Outcomes with Assessment Standards

for

Applied Mathematics 10

August 2002

This document is intended to assist teachers with the provincial implementation of Applied Mathematics 10.

This document replaces the February 2000 interim draft and the August 2001 revised draft, and reflects the 2002 Program of Studies.

Alberta
LEARNING
Learning and Teaching Resources Branch

The primary intended audience for this document is:

Administrators	
Counsellors	
General Audience	
Parent School Councils	
Parents	
Students	
Teachers	✓

ACKNOWLEDGEMENTS

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INTRODUCTION

Applied Mathematics 10 was available for optional implementation in September 1998. Teachers field validating Applied Mathematics 10 and preparing for provincial implementation in September 2000 expressed a need for a common understanding of the curriculum and assessment standards for this new course. In response to this need, and in keeping with Alberta Learning's goal of establishing and effectively communicating clear outcomes and high standards for each area of learning, this document of assessment standards was developed.

PURPOSE

Outcomes with Assessment Standards for Applied Mathematics 10 contains samples of tasks linked to the specific outcomes from the program of studies, along with information and commentaries about standards. Its purpose is to provide teachers of Applied Mathematics 10 with clearly stated standards to use as guidelines in their classroom instruction and assessment practices. This document is not intended as an assessment tool or test package, but only as a guide that provides models of tasks that can be assigned or discussed in the classroom. The sample tasks in this document are intended for teacher use, but also can be used to communicate to the broader educational community examples of acceptable student work and excellent student work in Applied Mathematics 10.

DEFINITIONS AND TERMINOLOGY

Standards

A *standard* is a reference point used in planning and evaluation. In evaluating educational performance, the following standards apply:

- *curriculum* and *assessment standards* apply to the assessment of individual students
- *achievement standards* apply to the assessment of student populations.

In this document, only curriculum and assessment standards are discussed.

Curriculum Standards

Curriculum standards are outcomes for a course or grade level of a program. The curriculum standards for Applied Mathematics 10 are defined by the general and specific outcomes outlined in the program of studies.

Outcomes

General outcomes are concise statements identifying what it is that students are expected to know and be able to do upon completion of a course or grade level of a program.

Specific outcomes are statements identifying the component knowledge, skills and attitudes of a general outcome. Specific outcomes identify a range of contexts in which the general outcomes apply.

Assessment Standards

Assessment standards are the criteria used for judging individual student achievement relative to the curriculum standards.

Assessment Instrument

An *assessment instrument* is a group of questions or tasks given to students to ascertain if they have met the requirements of the acceptable standard or the standard of excellence.

Blueprint

A *blueprint* is a classification of the questions or tasks making up a particular assessment instrument or an assessment plan. The classification may be in terms of standards (acceptable or excellent), content sections (topics or titles), mathematical understandings (concepts, procedures, problem-solving skills), or any other such classification.

Conceptual Understanding

Tasks assessing *conceptual understanding* are characterized by the use of such action verbs as defining, demonstrating, describing, developing, establishing, explaining, giving, identifying, illustrating, linking, recognizing and representing.

Procedural Knowledge

Tasks assessing *procedural knowledge* are characterized by the use of such action verbs as approximating, calculating, constructing, estimating, factoring, locating, rationalizing, simplifying, sketching, solving, using and verifying. Guided project work fits into this category.

Problem-solving Skills

Problem-solving skills refer to the construction of models describing applications in a broad range of contexts. They refer to the combination of algorithms to determine more complicated solutions and the use of appropriate technology to approximate difficult models or to carry out long calculation procedures. They also refer to the investigation of phenomena, the collection and interpretation of data in various contexts, and the transfer of knowledge from mathematics to other areas of human endeavour. Problem-solving tasks are characterized by the use of such action verbs as adapting, analyzing, combining, communicating, comparing, connecting, constructing, deriving, evaluating, fitting, investigating, justifying, modelling, proving, reconstructing and translating.

STANDARDS FOR APPLIED MATHEMATICS 10

The content specified in the program of studies for Applied Mathematics 10 contains the knowledge, skills and attitudes that all students are expected to acquire. As well, there are specific outcomes for problem solving and the use of technology. For complete details of the Applied Mathematics 10 course structure, refer to the program of studies.

Applied Mathematics 10 is designed to follow directly from Grade 9 Mathematics, so students taking Applied Mathematics 10 are presumed to have reached the acceptable standard or better in the outcomes of Grade 9 Mathematics. In particular, students will have facility in using the scientific calculator; have an ability to add, subtract and multiply polynomials; have an understanding of the Pythagorean theorem and the three primary trigonometric ratios; and have an ability to produce estimated lines of best fit for bivariate data.

The assessment standards for Applied Mathematics 10 include an acceptable and an excellent level of performance. Student performance should be measured on a range of tasks, some of which are routine and obvious tasks in familiar contexts and others which are nonroutine tasks in unfamiliar contexts.

Acceptable Standard

The *acceptable standard* of achievement in Applied Mathematics 10 is met by students who receive a course mark between, and including, 50 per cent and 79 per cent. Typically, these students have gained new skills and a basic knowledge of the concepts and procedures relative to the general and specific outcomes defined in the Applied Mathematics 10 program of studies. These students can apply this knowledge to a limited range of familiar problem contexts.

Standard of Excellence

The *standard of excellence* for achievement in Applied Mathematics 10 is met by students who receive a course mark at, or above, 80 per cent. Typically, these students have gained a breadth and depth of knowledge regarding the concepts and procedures, as well as the ability to apply this knowledge to a broad range of familiar and unfamiliar problem contexts. This standard signifies high-quality performance relative to the general and specific outcomes in the Applied Mathematics 10 program of studies.

Description of Standards

The following statements describe what is expected of Applied Mathematics 10 students who meet the *acceptable standard* or the *standard of excellence* on independent work. The statements represent the standards against which student achievement is measured.

Acceptable Standard	*Standard of Excellence*
Students who meet the *acceptable standard* in Applied Mathematics 10 consistently perform acceptable work on routine and obvious tasks in familiar contexts.	Students who meet the *standard of excellence* in Applied Mathematics 10 consistently perform excellent work on routine and obvious tasks in familiar contexts, and acceptable work on nonroutine tasks in unfamiliar contexts.

Acceptable Standard

These students have a basic understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in one or two ways, and can do simple conversions from one mode to another. They perform the mathematical operations and procedures that are fundamental to applied mathematics at the 10 level, and apply what they know in daily living contexts.

To meet the *acceptable standard*, students communicate about mathematical situations in an understandable way, using appropriate everyday and mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar contexts. These students adapt mathematical models from one situation to another.

Students meeting the *acceptable standard* are able to follow directions in such documents as calculator manuals, computer software documentation and measuring instrument manuals to perform familiar tasks. They know when to use such directions and can produce reliable results for familiar tasks.

Students meeting the *acceptable standard* have a positive attitude toward mathematics and a sense of personal competence in using mathematics. They demonstrate confidence when using common mathematical procedures and when applying problem-solving strategies in familiar settings.

Standard of Excellence

These students have a comprehensive understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in a variety of modes and can translate from one mode to another. They perform the mathematical operations and procedures that are fundamental to applied mathematics at the 10 level, apply what they know in daily living contexts and provide alternative solution procedures to verify results.

To meet the *standard of excellence,* students communicate about mathematical situations in a clear way, using numbers, diagrams and appropriate mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar and unfamiliar contexts. These students construct mathematical models by translating words into suitable numbers, diagrams, tables, equations and variables.

Students meeting the *standard of excellence* are able to follow directions in such documents as calculator manuals, computer software documentation and measuring instrument manuals to perform familiar and novel tasks. They know when to use and when to modify such directions, and can produce reliable results for both familiar and novel tasks.

Students meeting the *standard of excellence* have a positive attitude toward mathematics and show confidence in using mathematics meaningfully. They are self-motivated risk takers who persevere when solving novel problems. They take initiative in trying new methods and are creative in their approach to problem solving.

ASSESSMENT PLANS, SUGGESTED WEIGHTINGS AND BLUEPRINTS

These plans and blueprints provide general guidelines for the overall assessment of Applied Mathematics 10.

Suggested Weightings by Topic

Topic	Specific Outcomes (Summary)	Addison-Wesley Applied Mathematics 10 Resource	Suggested Per Cent Weighting
Measurement	1.1 Select and apply appropriate instruments, units of measure and measurement strategies to find lengths, areas and volumes. 1.2 Analyze the limitations of measuring instruments and measurement strategies, using the concepts of precision and accuracy. 1.3 Solve problems involving length, area, volume, time, mass and rates derived from these. 1.4 Interpret drawings, and use the information to solve problems. 1.5 Calculate the volume and surface area of a sphere, using formulas that are provided. 1.6 Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects.	Chapter 1 in Source Book Projects 1 and 2 in Project Book	18–20%
Number Patterns in Tables	2.1 Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively. 2.2 Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively. 2.3 Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system. 2.4 Use approximate representations of irrational numbers. 2.5 Communicate a set of instructions used to solve an arithmetic problem. 2.6 Perform arithmetic operations on irrational numbers, using appropriate decimal approximations. 2.7 Create and modify tables from both recursive and nonrecursive situations. 2.8 Use and modify a spreadsheet template to model recursive situations. 2.9 Solve problems involving combinations of tables.	Chapter 2 in Source Book Projects 3–7 in Project Book [To cover all of the specific outcomes in this topic, one of projects 6, 7 and 8 must be completed]	12–15%
Relations and Functions	3.1 Plot linear and nonlinear data, using appropriate scales. 3.2 Represent data, using function models. 3.3 Use a graphing tool to draw the graph of a function from its equation. 3.4 Describe a function in terms of ordered pairs, a rule or a graph. 3.5 Use function notation to evaluate and represent functions. 3.6 Determine the domain and range of a relation from its graph.	Chapter 3 in Source Book Project 9 in Project Book	18–20%
Line Segments	4.1 Solve problems involving distances between points in the coordinate plane. 4.2 Solve problems involving midpoints of line segments. 4.3 Solve problems involving rise, run and slope of line segments. 4.4 Solve problems using slopes of parallel lines and perpendicular lines.	Chapter 5 in Source Book Project 13 in Project Book	16–18%
Linear Functions	5.1 Rewrite linear expressions in terms of the dependent (responding) variable. 5.2 Determine the following characteristics of the graph of a linear function, given its equation: intercepts, slope, domain, range. 5.3 Determine the equation of a line, given information that uniquely determines the line. 5.4 Use variation and arithmetic sequences as applications of linear functions. 5.5 Determine the equation of a line of best fit, using: estimate of slope and one point, median–median method, least squares method with technology. 5.6 Use best-fit linear equations and their associated graphs to make predictions and solve problems. 5.7 Explain the significance of the parameters a and b in the best-fit equation $y = ax + b$. 5.8 Use technological devices to determine the correlation coefficient r. 5.9 Interpret the correlation coefficient r and its limitations for varying problem situations, using relevant scatterplots.	Chapter 6 in Source Book Projects 14–17 in Project Book	18–20%
Trigonometry	6.1 Solve problems involving two right triangles. 6.2 Extend the concepts of sine and cosine for angles from 0° to 180°. 6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems.	Chapter 7 in Source Book Projects 18–20 in Project Book	10–12%

Suggested Per Cent Weightings by Standard and Skill				
	Conceptual Understanding	Procedural Knowledge	Problem-solving Skills	Totals
Acceptable Standard	≅20%	≅30%	≅25%	75–80%
Standard of Excellence	≅10%	≅5%	≅10%	20–25%
Totals	25–30%	35–40%	30–35%	100%

TECHNOLOGY REQUIREMENTS

To meet the outcomes of Applied Mathematics 10, students will need access to a graphing calculator and a computer with a spreadsheet program. In addition, measuring devices in both metric and imperial units are required. In most classrooms, students will not use technology on a daily basis; however, there are some topics of study that require greater access than others. The following table contains descriptions of the technology-related knowledge and skills expected of students in each of the topic areas, with suggested time requirements.

Topic	Estimated Time Requirement for Access to Technology	Technology-related Knowledge and Skills
Measurement	3–4 hours	*Students will be expected to:* • use metric and imperial measurement devices for precision measurements • use a measurement conversion program on a graphing calculator
Number Patterns in Tables	5–10 hours	• use a computer spreadsheet program, including the application of user-defined functions

Relations and Functions	10–12 hours	• graph a scatterplot on a graphing calculator
		   – enter data in lists
		   – choose the appropriate window settings
		   – plot the graph
		• graph an equation, using a graphing calculator
		   – enter the equation in the $y =$ form
		• find the domain and range, using a graphing calculator
		   – display a table or use the trace function to estimate the domain and range

Line Segments	1–2 hours	• graph a scatterplot on a graphing calculator • use a calculator-based ranger (CBR) or a calculator-based laboratory (CBL) to collect data (optional) • use dynamic geometry software to explore the connections among rise, run, distance and slope (optional)

Linear Functions	10–12 hours	• find the slope and y-intercept, using a graphing calculator
		   – display a table or use the trace function to estimate the slope and y-intercept
		• find and graph the line of best fit for a data set, using a graphing calculator
		   – use the regression menu of a graphing calculator
		• determine the correlation coefficient for a data set, using a graphing calculator

Trigonometry	1–2 hours	• use a clinometer and transit to measure angles • use a graphing calculator to graph trigonometric functions

MATHEMATICAL CONVENTIONS USED IN THE DESCRIPTION OF SAMPLE TASKS

Approximations, Estimates and Exact Values

An *approximation* to an exact value is a decimal representation, correct to several decimal places, used in a problem. Usually these approximations result from calculator usage. For example, $\sqrt{3}$ may be replaced by the approximation 1.7321... during the computational phase of a problem.

An *estimate* of an exact value is an approximation that can be derived mentally and used as part of a mental calculation. For example, the conjecture $\sqrt{2} + \sqrt{8} = \sqrt{10}$ can be shown to be false by using the estimates 1.5, 2.8 and 3.1 for $\sqrt{2}, \sqrt{8}$ and $\sqrt{10}$.

The *exact value* of a rational number is of the form $\frac{p}{q}$, where p and q are integers. Unless the decimal representation terminates, the decimal representation will be an approximation. So $-\frac{5}{3}$ is exact, while $-1.666...$ is an approximation.

The *exact value* of an irrational number is expressed in such forms as $\sin 35°$, $\sqrt{3}$ or $\frac{\pi}{6}$. The decimal representation will always be an approximation. So $\sin 35°$ is exact, while 0.573576... is an approximation.

Calculator Use in Multipart Problems

The general rule is to keep the calculator running from beginning to end, using all the decimal places available on the calculator. All answers, both intermediate and final, are reported to the appropriate number of significant figures. However, rounded intermediate answers are **not** to be used as inputs into subsequent calculations. For example, a right-angled triangle with a measured hypotenuse of 11.9 cm and a shortest side of 6.1 cm would have the third side reported as 10.2 cm. However, if the length of the third side were used in the second part of a calculation, the more precise value 10.21763182... would be used as the input value.

Measured and Exact Input Data

If the *input data is measured*, all output data should be recorded in decimal form to the same quality as the least reliable part of the input data. For example, if the input data had 57.235 g for the mass and 17 mL for the volume, the density would be recorded as 3.4 g/mL with two significant digits. As a matter of convention, trigonometric problems with lengths to three significant digits yield angles to one-tenth of a degree, or one-hundredth of a radian.

If the *input data is exact,* all output data should be recorded in fractional form if at all possible. For example, the slope of the line joining the points (6, 11) and (17, 32) should be recorded as $\frac{21}{11}$, which is exact, rather than the decimal form 1.9090....

For Applied Mathematics 10, all trigonometric data can be assumed to be measured. In addition, most estimated regression coefficients used in calculations of correlation coefficients and predicted y-values can be assumed to be measured. All input data in coordinate geometry, algebraic expressions, relations and functions can be assumed to be exact. Most of the input data in number tables and number patterns can be assumed to be exact, except that money outputs are rounded to the nearest dollar or the nearest cent, depending on the context.

If there is any doubt whether any item of input data is exact or measured, assume it to be measured if it is part of an Applied Mathematics 10 problem.

STANDARDS LINKED TO OUTCOMES

The remainder of this document consists of several components that, taken together, provide guidance to teachers about the scope, depth and breadth of Applied Mathematics 10.

Topics

Topics are the mathematical themes that provide a convenient way for organizing concepts. The general and specific outcomes and the associated tasks in this part of the document are grouped according to topics.

General and Specific Outcomes

The general and specific outcomes for Applied Mathematics 10 identify what students are expected to know and be able to do upon completion of the course.

Strands

Strands are the underlying themes running throughout all of the mathematics programs. The Applied Mathematics 10 course includes:

- Number (Number Concepts)
- Number (Number Operations)
- Patterns and Relations (Relations and Functions)
- Shape and Space (Measurement)
- Shape and Space (3-D Objects and 2-D Shapes)
- Statistics and Probability (Data Analysis)

Mathematical Processes

These are the critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and to encourage lifelong learning in mathematics. One or more of the following mathematical processes is emphasized for each specific outcome.

[C]   Communication
[CN] Connections
[E]   Estimation and Mental Mathematics

[PS] Problem Solving
[R]   Reasoning
[T]   Technology
[V]   Visualization

More information on the general and specific outcomes, strands and mathematical processes can be found in the *Mathematics Applied and Pure Programs Program of Studies*.

Notes

Each specific outcome is followed by a note that provides additional information about the intended depth and breadth for that outcome. Teachers can refer to these for guidance when planning classroom activities and assessments that address the specific outcome at an appropriate level for the course.

Sample Tasks

Linked to each specific outcome are sample tasks. These consist of questions, problems or activities for classroom use with students. Teachers may use these directly or as models for developing their own tasks for group or independent work in the classroom, or as part of formative and summative assessment of students. The sample tasks and suggested solutions were developed and validated by classroom teachers of Applied Mathematics 10, but **they have not been validated with students**. They represent models of the kinds of questions and problems students should be able to solve, and the kinds of activities students should be able to perform to meet the specific outcome to which they are linked.

The following two examples illustrate the components expected in a student's written response to a problem, when a graphing calculator has been used to assist in determining the answer.

Example 1: When asked to solve a problem that asks for the coordinates of key points, from a description of a situation that can be expressed as a function, the following should be included in the student's response:

- the function in $y =$ form
- a sketch of the graph with the scales labelled
- the key points identified on the sketch
- a statement describing the relationship between the calculator screen output and the answer to the problem, within its context.

Example 2: When solving a problem that involves using the regression menu on a graphing calculator to determine the line of best fit for a data set, students can be expected to include the following in the written response:

- the data from input lists
- the regression formula used
- the equation of the line of best fit
- a sketch of the graph with the scales labelled
- a statement describing the relationship between the calculator screen output and the answer to the problem, within its context.

Descriptions of Student Performance (Related to Specific Outcome)

Each specific outcome is followed by a description of student performance at the acceptable standard and at the standard of excellence. These descriptions represent the judgements of practising classroom teachers in regard to how their students would perform on the particular specific outcome.

Descriptions of Student Performance (Related to Sample Tasks)

The sample tasks for a particular specific outcome are followed by a description of student performance at the acceptable standard and at the standard of excellence. These descriptions, though **not based on actual student responses**, represent the judgements of practising classroom teachers in regard to how their students would perform, given these particular tasks. For many of the sample tasks, it was presumed that students would have access to reference materials, such as calculator manuals, software documentation and the reference utility materials present in their textbooks.

STANDARDS IN MEASUREMENT

GENERAL OUTCOMES

- Use measuring devices to make estimates and to perform calculations in solving problems.
- Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

SPECIFIC OUTCOMES

1.1   Select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes. [E, PS, T]

1.2   Analyze the limitations of measuring instruments and measurement strategies, using the concepts of precision and accuracy. [C, R]

1.3   Solve problems involving length, area, volume, time, mass and rates derived from these. [C, E, PS]

1.4   Interpret drawings, and use the information to solve problems. [C, PS, V]

1.5   Calculate the volume and surface area of a sphere, using formulas that are provided. [CN, PS, V]

1.6   Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects; e.g., surface area to volume ratios. [CN, PS, R, V]

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.1   Select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes. [E, PS, T]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and	[T]	Technology
	Mental Mathematics	[V]	Visualization

Notes:

- Students are expected to measure small lengths to 0.01 mm or 0.001 in, using instruments supplied.
- The current technology favours vernier callipers and micrometers; however, alternative technologies can be used if they are available.
- Conversion between SI units and Imperial units should be required very rarely; e.g., when the problem input data are mixed—Imperial and SI.
- Proportions can be used to solve conversion problems algebraically. See specific outcome 1.6 for further details.
- This specific outcome is strongly linked to specific outcomes 1.3 and 1.4.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- use rulers, vernier callipers and micrometers to make length measurements - devise measurement strategies for simple measurements.	devise measurement strategies for complex measurements.

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General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.1 Select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes. [E, PS, T]

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Sample Tasks

Question:

1. Measure the outside diameter, the inside diameter and the wall thickness of a piece of PVC pipe that is supplied. Measure each of these quantities to the nearest 0.1 mm.

Solution:

1. Solutions will vary.

Question:

2. A person is 5'8" tall. What would their driver's licence report, given that the Motor Vehicles Branch uses whole numbers of centimetres?

Solution:

2. $\frac{5\ \text{feet}}{1\ \text{ft}} = \frac{n}{12\ \text{in}}$, $n = 60$ inches; so 5'8" = 68 inches

   $\frac{68\ \text{inches}}{1\ \text{inch}} = \frac{n}{2.54\ \text{cm}}$, $n = 172.73$ cm

   The driver's licence will report 173 cm.

[ ]	Conceptual	
[✓]	Procedural	
[✓]	Problem-solving	

Gencral and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.1 Select and apply appropriate instruments, units of measure (in SI and Imperial systems) and measurement strategies to find lengths, areas and volumes. [E, PS, T]

Question:

3. Materials: a two-dollar coin (toonie), a vernier calliper and a micrometer.

Task: Determine the ratio of the volumes of the metal components used in the construction of the toonie. Measure all relevant quantities to the nearest 0.1 mm, and explain the measurement strategy used.

Solution:

3. One possible solution is:
Thickness (height) = 1.5 mm
Outside diameter = 28.0 mm
Inside diameter = 16.9 mm
Volume of the inner gold-coloured portion (V_1)
$= \pi r^2 h$

$= \left(\pi \bullet \left(\frac{16.9}{2}\right)^2 \bullet 1.5 \right) \text{mm}^3$

$= 336.5 \text{ mm}^3$

Volume of the outer silver-coloured portion (V_2)
$= \pi r^2 h - V_1$

$= \left(\pi \bullet \left(\frac{28}{2}\right)^2 \bullet 1.5 \right) \text{mm}^3 - 336.5 \text{ mm}^3$

$= (923.6 - 336.5) \text{ mm}^3$
$= 587 \text{ mm}^3$
Ratio of volumes
336.5 : 587
or 1 : 1.7 (inner metal : outer metal)
Note: Answers will vary because dimensions may vary from coin to coin.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• measure all quantities in questions 1 and 3 to ± 0.1 mm • solve question 2 completely • calculate the volumes in question 3.	• calculate, correctly, the volumes and ratio in question 3 from measurements of the relevant dimensions • explain the measurement strategy used in question 3.

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General and Specific Outcomes

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General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.2   Analyze the limitations of measuring instruments and measurement strategies, using the concepts of precision and accuracy. [C, R]

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[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and	[T]	Technology
	Mental Mathematics	[V]	Visualization

Notes:

- A detailed treatment of significant figures is not necessary.
- Students need an understanding that the accuracy of output data can be no greater than the accuracy of the input data.
- The distinction between precision and accuracy is difficult for students.
- These concepts are explored further in Applied Mathematics 20.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- select the best instrument or strategy for making a particular measurement - make obvious improvements to a measuring instrument.	- justify the use of a particular measuring instrument or strategy, based on the concepts of precision and accuracy - make and justify improvements in measuring instruments and measurement strategies.

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.2 Analyze the limitations of measuring instruments and measurement strategies, using the concepts of precision and accuracy. [C, R]

Question:

1. Which of the following three methods of measuring would be the most suitable in measuring the length of a parking lot: pacing, trundle wheel or metre stick? Why? Under what conditions might this not be the most suitable method? Explain why.

Solution:

1. Use the trundle wheel. It measures to 10 cm, and will give four significant digits for parking lots between 100 m and 1000 m in length. The metre stick will give six significant digits, but it will take a far longer time to complete the measurements. A trundle wheel may not be suitable if the parking lot is uneven or contains many obstructions. The wheel may jump, stick or slip.

Question:

2. Explain why a micrometer is more precise than a ruler in measuring the thickness of five sheets of thin metal.

Solution:

2. The micrometer is more precise for measuring small lengths, as it can measure to 0.01 mm, whereas a ruler measures to 1 mm.

Question:

3. Which method is more accurate for determining the volume of a marble: using vernier callipers and a volume formula or using a graduated cylinder and the water displacement method? Why?

Solution:

3. In most circumstances, using a vernier calliper and a volume formula will be more accurate. Usually the vernier calliper will give the diameter to 0.1 mm, and the volume to 1 mm^3. Water displacement will give the volume to 0.1 mL, or 100 mm^3.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• determine that the trundle wheel would be the most suitable measuring tool, and explain why in question 1 • explain that the micrometer uses smaller units than a ruler and is more precise, in question 2 • state that using vernier callipers and a volume formula is more accurate for determining the volume than using the displacement method, in question 3.	• state under what conditions the wheel would not be the most suitable tool in question 1, and explain why • state that a ruler measures only to the nearest mm, whereas a micrometer can measure to the nearest 0.01 mm, in question 2 • provide a detailed explanation, in question 3, of why using vernier callipers and a volume formula to calculate the volume is more accurate than using the displacement method, since the input data for the displacement method is to the nearest 0.1 mL and for the vernier calliper is to the nearest 0.1 mm.

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.3 Solve problems involving length, area, volume, time, mass and rates derived from these. [C, E, PS]

[C] Communication   [PS] Problem Solving
[CN] Connections   [R] Reasoning
[E] Estimation and   [T] Technology
     Mental Mathematics   [V] Visualization

Notes:

- Specific outcomes 1.3 and 1.4 can be linked, and many problems can cover both specific outcomes.
- Students have learned the perimeter, area and volume formulas for geometric figures in grades 7, 8 and 9 mathematics.
- Examples of derived rates are speed, density and unit cost.
- Both simple rates and combined rates are included in this specific outcome.
- Students should have experience with problems where the formula is given, as well as with problems where the formula is not given.
- Students should be encouraged to draw diagrams or sketches when solving problems.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- solve problems involving simple areas and volumes, including drawing relevant diagrams - solve problems involving composite areas and volumes, if relevant diagrams are given - substitute input data into formulas describing areas and volumes - calculate a simple rate from given input data.	- solve problems involving composite areas and volumes, including drawing relevant diagrams - calculate input data from a given rate - combine rates in a given context.

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General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.3 Solve problems involving length, area, volume, time, mass and rates derived from these. [C, E, PS]

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[   ] Conceptual
[✓] Procedural
[✓] Problem-solving

Question:

1. An athletic running track has two straight sections and two ends that are exact semicircles. The straight sections are each 105 m long and the ends have a diameter of 65 m.
   a. Calculate the perimeter of the track.
   b. Calculate the area enclosed by the track.

Solution:
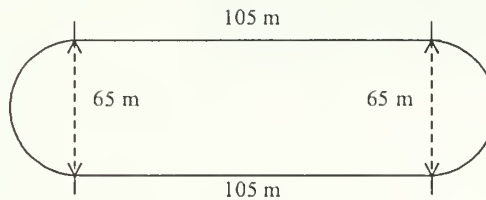
1. a. Perimeter = $(105 \text{ m}) + (105 \text{ m}) + (\pi)(65 \text{ m})$
   $= (210 + 204.2)\text{m}$
   $= 414 \text{ m}$

   b. Area = rectangle + 2 semicircles
   $= (105 \text{ m})(65 \text{ m}) + \pi (32.5 \text{ m})^2$
   $= (6825 + 3318) \text{ m}^2$
   $= 10 \; 143 \text{ m}^2$
   $\doteq 10 \; 100 \text{ m}^2$

Question:

2. Which represents the higher average speed: running a mile in four minutes or running 1500 m in 3 min 30 s? (One mile is 1609 m.)

Solution:

2. Four minute mile $= \dfrac{1609 \text{ m}}{240 \text{ s}} = 6.70 \text{ m/s}$

   3 min 30 s for 1500 m $= \dfrac{1500 \text{ m}}{210 \text{ s}} = 7.14 \text{ m/s}$

   The 1500 m speed is 0.44 m/s faster.

(continued)

Sample Tasks
[   ] Conceptual
[   ] Procedural
[✓] Problem-solving

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.3   Solve problems involving length, area, volume, time, mass and rates derived from these. [C, E, PS]

(continued)

Question:

3.   The density of pure gold is 19.3 g/cm³. Verify if a "gold brick" of mass 454 g, measuring 10 cm by 1.8 cm by 1.3 cm, is pure gold. (Density = mass/volume.)

Solution:

3. $D = \frac{m}{V}$

$$= \frac{454 \text{ g}}{10 \text{ cm} \times 1.8 \text{ cm} \times 1.3 \text{ cm}}$$

$$= \frac{454 \text{ g}}{21.4 \text{ cm}^3}$$

$$= 19.4 \text{ g/cm}^3$$

The brick is likely pure gold, given measurement errors due to rounding off.

Question:

4.   What is your average speed on a trip when you travelled 20 km at 60 km/h and 165 km at 110 km/h?

Solution:

4.   Total distance of trip is 185 km.
Total time taken is 20 km/(60 km/h) + 165 km/(110 km/h) = 1.83 h

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{185 \text{ km}}{1.83 \text{ h}} = 101 \text{ km/h}$$

(continued)

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.3 Solve problems involving length, area, volume, time, mass and rates derived from these. [C, E, PS]

[   ]   Conceptual
[   ]   Procedural
[✓]   Problem-solving

(continued)
Question:

5. Rewrite the formula for the volume of a sphere, $V = \frac{4}{3}\pi r^3$, in terms of the diameter d.

Solution:

5. $d = 2r, r = \frac{d}{2}$.

   So $V = \frac{4}{3}\pi r^3$ is rewritten as

   $V = \frac{4}{3}\pi r^3 \left(\frac{d}{2}\right)^3$ or $V = \frac{4}{3}\pi \left(\frac{d^3}{8}\right)$

   So $V = \frac{1}{6}\pi d^3$.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• solve question 1, if the diagram is given • solve question 2 • provide a complete solution to question 3 • interpret question 4 as a straight average problem • recognize that $r = \frac{d}{2}$ in question 5.	• solve question 1, if the diagram is not given • provide a complete solution to question 4 • provide a complete solution to question 5.

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.4 Interpret drawings, and use the information to solve problems. [C, PS, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Specific outcomes 1.3 and 1.4 can be linked, and many problems can cover both specific outcomes.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• read input data in metric or Imperial units from drawings • use data from drawings as input into formulas.	• infer input data from drawings • make additional drawings as needed to solve the problem • rewrite formulas to correspond to dimensions on drawings.

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.4 Interpret drawings, and use the information to solve problems. [C, PS, V]

Question:

1. The diagram shows the cross section of a cylindrical pipe. The pipe walls are 1.0 mm thick.
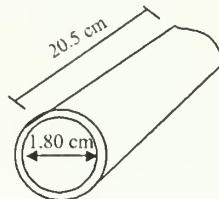
a. What is the outside diameter of the pipe?
b. What volume of water, to the nearest mL, can be stored in the pipe?
c. What volume of metal, to the nearest cm^3, is needed to construct the pipe?

Solution:

1. a. 1.80 cm + 2(0.10 cm)
   = 2.00 cm
   The outside diameter is 2.00 cm.

   b. $V = \pi r^2 \ell$

   $V = \pi (0.90 \text{ cm})^2 (20.5 \text{ cm})$

   $V = 52 \text{ cm}^3$

   The volume of water that can be stored in the pipe is 52 mL.

   c. $V = V_1 - V_2$
   where V_1 = volume of outside cylinder
   V_2 = volume of inside cylinder = 52 cm^3
   $\therefore V = \pi [1.00 \text{ cm}]^2 [20.5 \text{ cm}] - 52 \text{ cm}^3$
   = 12 cm^3 of metal.
   The volume of metal needed to construct the pipe is 12 cm^3.

(continued)

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.4   Interpret drawings, and use the information to solve problems.
       [C, PS, V]

(continued)
Question:

2.   A person is building a greenhouse in the shape of a pyramid, as shown in the diagrams below. The vertex *V* is 18.0 feet above the floor.

Top View   Side View
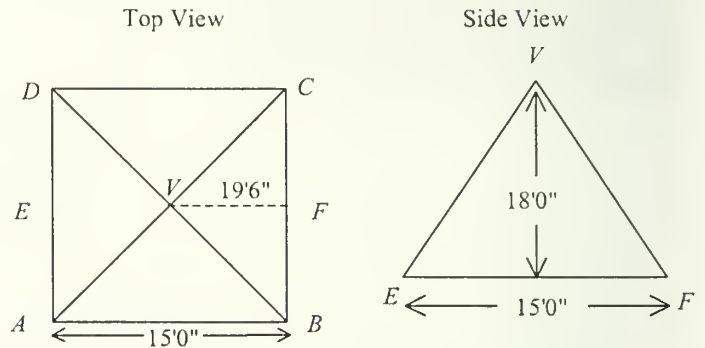
a.   What is the area of one wall?
b.   What is the surface area of glass required, if all four walls are constructed from glass?
c.   If each pane of glass is 1'6" × 1'6", what is the minimum number of panes of glass, allowing for no waste, that need to be purchased to build the greenhouse?
d.   If the floor is to be constructed from concrete that is 4.00 inches thick, what is the volume of concrete, in ft³, required to build the floor?
e.   Fans are required to completely circulate the air inside the greenhouse. The volume of air in the building determines the number of fans required. What volume of air needs to be circulated?
f.   If one fan is sufficient to circulate 375 ft³ of air, how many fans does the greenhouse require?
g.   When all the fans are operating, it takes 1.5 h to circulate the entire volume of air. How long would it take to circulate the air if only one fan was operating?

(continued)

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.4 Interpret drawings, and use the information to solve problems. [C, PS, V]

(continued)
Solution:

2. a. Area $VAB = \dfrac{1}{2} bh = \dfrac{1}{2} (15.0 \text{ ft})(19.5 \text{ ft}) = 146 \text{ ft}^2$ (rounded off)

   b. Total area = $4(146.25 \text{ ft}^2) = 585 \text{ ft}^2$

   c. $\dfrac{585 \text{ ft}^2}{(1.5 \text{ ft})^2/\text{pane}}$ = 260 panes; it would require 260 panes, assuming no waste.

   d. Volume of concrete = $Ah = (15.0 \text{ ft})^2(\dfrac{1}{3} \text{ ft}) = 75 \text{ ft}^3$

   e. Volume of air = $\dfrac{1}{3} Ah = \dfrac{1}{3}(15.0 \text{ ft})^2(18.0 \text{ ft}) - 1350 \text{ ft}^3$

   f. Number of fans = $\dfrac{1350 \text{ ft}^3}{375 \text{ ft}^3} = 3.6$; so four fans are needed.

   g. With one fan, it would take 1.5 h × 4 = 6 h to circulate the air.

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General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.4   Interpret drawings, and use the information to solve problems.
[C, PS, V]

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(continued)
Question:

3.   A conductor's platform is to be covered with a carpet. The diagrams below show the platform from the top and the side.
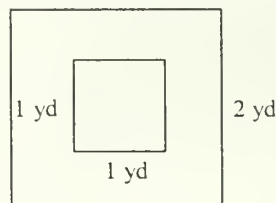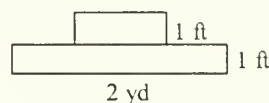
a.   Determine the surface area of **all** visible portions of the platform.
b.   Carpet is sold in rolls that are 4 yd wide. What is the minimum length, to the nearest whole yard, required to cover the visible surfaces of the platform?
c.   Draw a diagram of your cutting pattern. What is the area of the wasted carpet?
d.   If carpet costs $14.99/yd^2, how much money would be saved if there had been no waste?

Source:

Manitoba Education Senior 2 Applied Mathematics exercises, Page E–14. Reproduced with permission.

Solution:

3.   a.   Surface area is the sum of:
$$2 \text{ yd} \times 2 \text{ yd} = 4 \text{ yd}^2$$
$$4 \times 1 \text{ yd} \times 0.3 \text{ yd} = 1.2 \text{ yd}^2$$
$$\text{and} \quad 4 \times 2 \text{ yd} \times 0.3 \text{ yd} = \underline{2.4 \text{ yd}^2}$$
$$\text{surface area} = 7.6 \text{ yd}^2$$

b. $7.6 \text{ yd}^2 \div 4 \text{ yd} = 1.9 \text{ yd}$
$$\approx 2 \text{ yd long}$$
The carpet should be cut 2 yd long.

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General and Specific Outcomes

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<tr><td colspan="2">

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

1.4 Interpret drawings, and use the information to solve problems. [C, PS, V]

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(continued)
Solution:

3.   c.   A possible diagram:

| | 2 yd × 2 yd | 0.3 yd × 2 yd | 0.3 yd × 2 yd | 0.3 yd × 2 yd | 0.3 yd × 2 yd | 0.3 yd × 1 yd / 0.3 yd × 1 yd | 0.3 yd × 1 yd / 0.3 yd × 1 yd | waste material |

2 yd
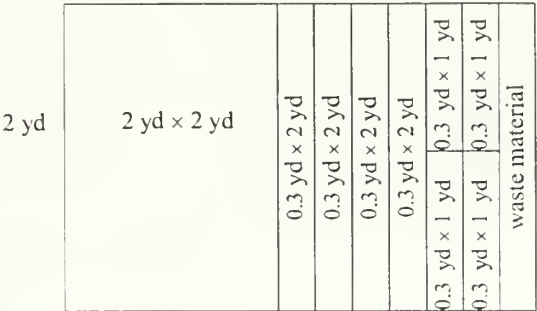
The waste material is 0.2 yd × 2 yd = 0.4 yd^2.

d.   Waste cost is 0.4 yd^2 × \$14.99/yd^2 = \$6.00.
The savings would be \$6.00.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• complete question 1, part a • complete question 1, part b, if the formula is given • calculate the area of one side of the pyramid in question 2 • calculate the total surface area of the four sides in question 2 • calculate the volume of the concrete floor in question 2 • calculate the volume of air in the pyramid, in question 2, when given the formula • calculate the number of fans needed in question 2 • determine the surface area of the visible portions of the platform in question 3 • determine the minimum length of carpet required to cover the visible surfaces in question 3 • provide a general sketch of the cutting pattern in question 3.	• complete question 1, part b, if the formula is not given • complete question 1, part c • determine the number of panes of glass required in question 2 • calculate the volume of air in the pyramid, in question 2, without the formula being given • calculate the number of fans needed in question 2, correctly rounding up • calculate the length of time it takes for one fan to circulate the air in question 2 • determine the area and cost of the wasted carpet in question 3 • draw an accurate diagram of the cutting pattern in question 3.

General and Specific Outcomes

General Outcome

Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Specific Outcome

1.5 Calculate the volume and surface area of a sphere, using formulas that are provided.
[CN, PS, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and	[T]	Technology
	Mental Mathematics	[V]	Visualization

Notes:

- This specific outcome should be strongly linked to specific outcomes 1.1 and 1.4.
- The connection between radius and diameter is an essential part of this specific outcome.
- Diameters and radii should be measured when at all possible.
- The displacement method can be used to find volumes of spheres.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- use given formulas and dimensions to calculate the volume and surface area of a sphere - determine the volume of a sphere in mL or cm³.	determine the radius or diameter of a sphere, using an appropriate formula.

General and Specific Outcomes	Sample Tasks

General and Specific Outcomes

General Outcome

Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Specific Outcome

1.5   Calculate the volume and surface area of a sphere, using formulas that are provided.
[CN, PS, V]

Sample Tasks

[   ]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

Question:

1.   Given the formula $V = \frac{4}{3}\pi r^3$, calculate the volume of a sphere with diameter 2½ inches.

Solution:

1. $V = \frac{4}{3}\pi(1.25\text{ in})^3$
     $= 8.2\text{ in}^3$

Question:

2.   Given the formula $SA = 4\pi r^2$, calculate the surface area of a sphere with radius 10.0 cm.

Solution:

2. $SA = 4\pi(10.0\text{ cm})^2 = 1260\text{ cm}^2$

Question:

3.   a.   A student used the displacement method to determine the volume of a sphere. Her experimental results were 13 mL, 15 mL, 14 mL and 14 mL. What is the volume of the sphere in cm³?
     b.   Determine the radius or diameter of the sphere, using an appropriate formula.

Solution:

3.   a.   Average volume displaced is 14 mL; therefore, the volume of the sphere is 14 cm³.
     b.   Method 1 (substituting numerical values first, and then solving the equation).
          If V is measured in cm³,

$$14\text{ cm}^3 = \frac{4}{3}\pi r^3$$
$$42 = 4\pi r^3$$
$$\frac{42}{4\pi} = r^3$$
$$r = \sqrt[3]{\frac{42\text{ cm}^3}{4\pi}}$$
$$r = 1.5\text{ cm}$$

(continued)

General and Specific Outcomes	Sample Tasks

General and Specific Outcomes

General Outcome

Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Specific Outcome

1.5 Calculate the volume and surface area of a sphere, using formulas that are provided.
[CN, PS, V]

Sample Tasks

[   ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)

Solution:

3. b. Method 2 (rearranging formula for r before substituting)

$$V = \frac{4}{3}\pi r^3$$

$$3V = 4\pi r^3$$

$$\frac{3V}{4\pi} = r^3$$

$$r = \sqrt[3]{\frac{3V}{4\pi}}$$

$$r = \sqrt[3]{\frac{3(14)}{4\pi}} = 1.5 \text{ cm}$$

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• solve questions 1 and 2 • determine the volume in cm³ for question 3, part a.	• determine the radius in question 3, part b.

General and Specific Outcomes

General Outcome

Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Specific Outcome

1.6 Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects; e.g., surface area to volume ratios. [CN, PS, R, V]

[C]   Communication   [PS] Problem Solving
[CN] Connections   [R]   Reasoning
[E]   Estimation and   [T]   Technology
     Mental Mathematics   [V]   Visualization

Notes:

- This specific outcome should be strongly linked to specific outcomes 1.1 and 1.4.
- Students should be able to solve for the unknown in $\frac{a}{b} = \frac{c}{d}$, and students should be able to use proportions.
- Similar figures have the same linear scale factor in each direction.
- Cases where there are different scale factors for length, width and height are studied in Applied Mathematics 20 and Applied Mathematics 30.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- determine linear scale factors from length measures - find area and volume scale factors from linear scale factors - calculate unknown lengths from scale factors.	- find areas and volumes, given the linear scale factors - find linear scale factors from area or volume scale factors.

General and Specific Outcomes

General Outcome

Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Specific Outcome

1.6 Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects; e.g., surface area to volume ratios. [CN, PS, R, V]

Sample Tasks

[   ]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

Question:

1. a. Using a computer or a photocopier, increase the length and width of a 4" by 6" rectangular picture by 25%, and compare the linear scale factor with the area scale factor. Explain why they are not the same.
    b. Predict what the new linear scale factor would be, if the area of the original picture was increased by 35%.

Solution:

1. a. The dimensions of the enlarged picture are 5" by 7½". The linear scale factor is 1.25 : 1.
        The area of the enlarged picture is 37.5 in^2, and the area of the original is 24 in^2. The area scale factor is 1.56 : 1.
        Each dimension of the rectangle is multiplied by 1.25; so the area, which is length times width, is multiplied by 1.25 twice.
    b. The new area is 1.35×24 in^2 = 32.4 in^2. Let x be the linear scale factor. New dimensions are $4x" \times 6x" = 24x^2$.
        Therefore, $24x^2 = 32.4$ in^2

        $$x = \sqrt{\frac{32.4}{24}}$$
        $$= 1.16$$
        The new linear scale factor would be 1.16 : 1.
        OR
        If L^2 is the area scale factor
        then $L^2 = 1.35 : 1$
        and $L = \sqrt{1.35}$: 1
        = 1.16 : 1
        The new linear scale factor would be 1.16 : 1.

Question:

2. On a simplified provincial map, Town A and Town B are labelled such that Town A is 300 km directly north of Town B (6 cm on the map).
    a. Determine the linear scale factor.
    b. Determine the straight line distance, in kilometres, from the northern border to the southern border of the province, if the distance on the map is 24 cm.
    c. If the area of the province on the map is 258 cm^2, what is the area of the province in km^2?
    d. What is the area scale factor?

Solution:

2. a. 6 cm : 300 km = 1 cm : 50 km or 1 : 5 000 000
    b. $24 \text{ cm} \times \frac{300 \text{ km}}{6 \text{ cm}} = 1200 \text{ km}$

        OR

        $\frac{x}{24 \text{ cm}} = \frac{300 \text{ km}}{6 \text{ cm}}$
        $x = 1200$ km

(continued)

General Outcome

Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Specific Outcome

1.6   Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects; e.g., surface area to volume ratios. [CN, PS, R, V]

Sample Tasks

[   ]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

(continued)
Solution:

2.   c. $258 \text{ cm}^2 \times \left(\frac{300 \text{ km}}{6 \text{ cm}}\right)^2 = 645\ 000 \text{ km}^2$

   OR

   $\left(\frac{x}{258 \text{ cm}^2}\right) = \left(\frac{300 \text{ km}}{6 \text{ cm}}\right)^2$

   $x = \left[\frac{(300 \text{ km})^2}{36 \text{ cm}^2}\right] \bullet \left[258 \text{ cm}^2\right]$

   $x = 64\ 500 \text{ km}^2$

   d. $(6 \text{ cm} : 300 \text{ km})^2 = 1 \text{ cm}^2 : 2500 \text{ km}^2$

Question:

3.   Cube *A* has dimensions 4 cm × 4 cm × 4 cm, and Cube *B* has dimensions 12 cm × 12 cm × 12 cm.
   a.   What is the linear scale factor between Cube *A* and Cube *B*?
   b.   Compare the surface area and volume of each.
   c.   What is the ratio of the surface areas of the two cubes?
   d.   What is the ratio of the volumes of the two cubes?
   e.   Explain the general relationships between the surface area ratio and the linear scale factor, and the volume ratio and the linear scale factor.
   f.   Do these relationships apply to other geometric shapes like spheres and cylinders?

Solution:

3.   a.   The linear scale factor is 1 : 3.
   b.

Cube	Surface Area (cm^2)	Volume (cm^3)
A	96	64
B	864	1728

   c.   surface area ratio 96 : 864 = 1 : 9

   $\frac{A(\text{Cube } B)}{A(\text{Cube } A)} = (\text{linear scale factor})^2$

   $\frac{A(\text{Cube } B)}{A(\text{Cube } A)} = \left(\frac{3}{1}\right)^2$

   $\therefore \frac{A(\text{Cube } B)}{A(\text{Cube } A)} = \frac{9}{1}$

(continued)

General and Specific Outcomes	Sample Tasks

General and Specific Outcomes

General Outcome

Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Specific Outcome

1.6   Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects; e.g., surface area to volume ratios.
[CN, PS, R, V]

(continued)
Solution:

3.   d.   volume ratio $64 : 1728 = 1 : 27$

$$\frac{V(\text{Cube } B)}{V(\text{Cube } A)} = (\text{linear scale factor})^3$$

$$\frac{V(\text{Cube } B)}{V(\text{Cube } A)} = \left(\frac{3}{1}\right)^3$$

$$\frac{V(\text{Cube } B)}{V(\text{Cube } A)} = \frac{27}{1}$$

e.   Square the linear scale factor to get the surface area ratio, and cube the linear scale factor to get the volume ratio.

f.   These relationships apply to all geometric shapes and solids, not just cubes.

Question:

4.   a.   Use $V = \frac{1}{6}\pi d^3$ to determine the volume of Sphere A, which has a diameter of 3 cm, and Sphere B, which has a diameter of 6 cm.

b.   Determine the scale factors of the diameters and the volumes of Sphere A and Sphere B. Explain why these scale factors are not the same.

Solution:

4.   a. $V(A) = 14.1 \text{ cm}^3$ $V(B) = 113 \text{ cm}^3$

b.   The scale factor of the diameters is $1 : 2$.
The scale factor of the volumes is $1 : 8$.
The volume has a $1 : 2$ increase in length, width and height; **or**, using $V = \frac{1}{6}\pi d^3$, the $1 : 2$ scale factor for diameter is cubed to obtain $1 : 8$.

(continued)

General and Specific Outcomes

General Outcome

Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Specific Outcome

1.6   Determine the relationships among linear scale factors, areas, the surface areas and the volumes of similar figures and objects; e.g., surface area to volume ratios. [CN, PS, R, V]

Sample Tasks

[   ]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

(continued)

Question:
5.   A model house includes a miniature pool that holds 24 cm³ of water. The actual pool holds 81 000 L. Determine the scale factor of the model.

Solution:
5.   Since $SF^3 = \frac{V(\text{Image})}{V(\text{Original})}$

Then $SF = \sqrt[3]{\frac{V(I)}{V(O)}}$

$= \sqrt[3]{\frac{24 \text{ mL}}{81000 \text{ L} \times 1000 \text{ mL}}}$

$= 0.00666\ldots$

$= \frac{1}{150}$ or $1 : 150$

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- determine the linear scale factor and complete the area scale factor for question 1 - determine the linear scale factor and the distance from the northern border to the southern border for question 2 - calculate the ratio of the surface areas and the ratio of the volumes of the two cubes in question 3 - determine the volumes of the spheres in question 4.	- explain why the linear scale factor and the area scale factor for question 1 are not the same - predict the linear and area scale factors for the per cent increases in question 1 - determine the area scale factor and the area of the province for question 2 - explain the general relationships in question 3, and apply the relationships to other geometric shapes - determine the scale factors of the volumes and the diameters of the two spheres in question 4 - explain why the scale factors of the diameters and the volumes are not the same in question 4 - solve question 5 completely.

STANDARDS IN NUMBER PATTERNS IN TABLES

GENERAL OUTCOMES

- Analyze the numerical data in a table for trends, patterns and interrelationships.
- Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.
- Use basic arithmetic operations on real numbers to solve problems.
- Describe and apply arithmetic operations on tables to solve problems, using technology as required.

SPECIFIC OUTCOMES

2.1 Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively (not calculated from previous data). [C, CN]

2.2 Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively (calculated from previous data). [C, CN]

2.3 Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system. [C, R, V]

2.4 Use approximate representations of irrational numbers. [R, T]

2.5 Communicate a set of instructions used to solve an arithmetic problem, including instructions that follow the algebraic format of the formula. [C]

2.6 Perform arithmetic operations on irrational numbers, using appropriate decimal approximations. [E, T]

2.7 Create and modify tables from both recursive and nonrecursive situations. [PS, T, V]

2.8 Use and modify a spreadsheet template to model recursive situations. [PS, T, V]

2.9 Solve problems involving combinations of tables, using:
- addition or subtraction of two tables
- multiplication of a table by a real number
- algebraic processes to build spreadsheet functions and templates.
[PS, T, V]

Strand: Number (Number Concepts)

Students will:

- use numbers to describe quantities
- represent numbers in multiple ways.

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

General and Specific Outcomes

General Outcome

Analyze the numerical data in a table for trends, patterns and interrelationships.

Specific Outcome

2.1 Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively (not calculated from previous data).
[C, CN]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and	[T]	Technology
	Mental Mathematics	[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 1.1.
- Algebraic expressions can include expressions that use variables and expressions that describe formulas from spreadsheets.
- Teachers may need to define terms that are commonly used as headers in tables.
- Units, except for $, should be placed in column headers and not in table entries.
- Technology in the form of spreadsheets may be used to address this specific outcome, but the use of technology is not mandatory.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• obtain specific information from a row/column of a table • use words and one-step algebraic expressions to describe the data and the interrelationships between two rows or two columns of a table.	• use multistep algebraic expressions in tables • use words to describe the interrelationships among multiple rows or columns of a table • predict and justify answers.

General and Specific Outcomes

General Outcome

Analyze the numerical data in a table for trends, patterns and interrelationships.

Specific Outcome

2.1   Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are not related recursively (not calculated from previous data).
[C, CN]

Sample Task

[✓]   Conceptual
[   ]   Procedural
[✓]   Problem-solving

Question:

1.   Magazine publishers are examining sales for the month of September. Below is a table with a record of the results.

Magazine Title	Sales	Price	Revenue
Teen Matters	127	$3.95	$501.65
YZ	132	$3.25	$429.00
Big Trucks	98	$5.25	$514.50
Fitness	158	$5.95	$940.10
Get Real	276	$4.65	$1283.40

   a.   How many magazines were sold in September?
   b.   What is the total revenue for all magazines?
   c.   Describe how to calculate the revenue for each magazine, in words and as a formula.
   d.   The publishers are changing the price of *YZ* to $3.75. What is the projected revenue of *YZ* magazine for the month of October, if sales remain the same?
   e.   Do you think that *YZ* sales will increase or decrease after the price increase? What effect will this have on *YZ* revenue? Support your answer with calculations.

Solution:

1.   a.   The total number of magazines sold was 791.
   b.   The total revenue was $3668.65.
   c.   Multiply the number of magazines purchased by the price per magazine.
        Formula: $R = S \times P$
   d.   The revenue for *YZ* will be 132 × $3.75 or $495.00 in October, assuming no change in sales.
   e.   Answers will vary. Students should be expected to justify both their prediction and their calculations.

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	**A student demonstrating the *standard of excellence* can also:**
- calculate the total number of magazines sold, using addition - calculate the total revenue, using addition - describe how to calculate revenue in words - determine the revenue in part d - provide an incomplete answer to part e, often limited to an unsupported guess.	- describe how to calculate revenue by devising an appropriate formula - apply the formula to determine the revenue in part d - make predictions for part e, based on personal experience; justify the predictions; and support the predictions with numerical data.

General and Specific Outcomes

General Outcome

Analyze the numerical data in a table for trends, patterns and interrelationships.

Specific Outcome

2.2 Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively (calculated from previous data). [C, CN]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 1.2.
- Algebraic expressions can include expressions that use variables and expressions that describe formulas from spreadsheets.
- Teachers may need to define terms that are commonly used as headers in tables.
- Units, except for $, should be placed in column headers and not in table entries.
- Technology in the form of spreadsheets could be used in addressing this specific outcome, but are not required. Students are not expected to create spreadsheets for this specific outcome.
- Although questions involving compound interest are a good application of this specific outcome, questions should be restricted to those involving interest that is compounded annually.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• use words and one-step algebraic expressions to describe the data and the interrelationships between two rows of a table.	• use multistep algebraic expressions in tables • use words to describe interrelationships among multiple rows or columns of a table • predict and justify answers.

<table>
<tr><td colspan="2" align="center">**General and Specific Outcomes**</td></tr>
<tr><td colspan="2">**General Outcome**</td></tr>
<tr><td colspan="2">Analyze the numerical data in a table for trends, patterns and interrelationships.</td></tr>
<tr><td colspan="2">**Specific Outcome**</td></tr>
<tr><td>2.2</td><td>Use words and algebraic expressions to describe the data and the interrelationships in a table with rows that are related recursively (calculated from previous data). [C, CN]</td></tr>
</table>

Question:

1. The following table contains information about a loan.

	A	B	C	D	E	F
1	Year	Opening Balance	Interest Rate (%)	Interest Charged	Annual Payment	Closing Balance
2	1	$20 000.00	5	$1000.00	$4619.50	$16 380.50
3	2	$16 380.50	5	$819.03	$4619.50	$12 580.03
4	3	$12 580.03	5	$629.00	$4619.50	$8 589.53
5	4	$8 589.53	5	$429.48	$4619.50	$4 399.51
6	5	$4 399.51	5	$219.98	$4619.50	$0.00

    a.   How long is the term of the loan?
    b.   What is the total interest for the term?
    c.   Describe how the bank calculated the amounts in D2 and F2.

Solution:

1.   a.   The term is 5 years.
        b.   The total interest is $3097.49.
        c.   D2 = B2 × 0.05 or D2 = B2 ∗ C2/100 (using spreadsheet notation)
            F2 = B2 + D2 – E2

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• interpret the table to correctly determine the term of the loan and the total interest • provide partial descriptions of how to determine columns D and F.	• provide complete descriptions of how to determine columns D and F.

General and Specific Outcomes

General Outcome

Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.

Specific Outcome

2.3   Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system. [C, R, V]

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 5.1.
- Classification of numbers includes criteria for inclusion of the number into the appropriate subset of the real number system.
- This topic was introduced in Grade 9 Mathematics.
- This specific outcome can be addressed throughout the curriculum. Teachers may choose to address this outcome within the topics of Measurement, Relations and Functions, and Trigonometry.
- It may be beneficial to discuss this specific outcome in conjunction with specific outcome 3.6.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• categorize numbers in the appropriate number system • explain how a number can belong to more than one number set.	• describe the different number sets.

General and Specific Outcomes

General Outcome

Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.

Specific Outcome

2.3 Classify numbers as natural, whole, integer, rational or irrational, and show that these number sets are nested within the real number system. [C, R, V]

Question:

1. a. Classify the following numbers into the appropriate number set:
      $8, \sqrt{5}, 0, \frac{5}{9}, -6$.

      natural _____   rational _____
      whole _____   irrational _____
      integer _____

   b. Explain why 8 belongs to four number sets.

Solution:

1. a. natural 8   rational $8, 0, \frac{5}{9}, -6$

      whole 8, 0   irrational $\sqrt{5}$
      integer 8, 0, –6

   b. The number 8 is a natural number, so must also be a whole number and an integer. It is also a rational number as it can be written as the fraction $\frac{8}{1}$.

Question:

2. Present an analogy from the real world that is like the relationship among natural numbers, whole numbers, integers, rational numbers and real numbers.

Solution:

2. Answers will vary, but one example is the biological classification system: species, genus, family, order, class. Another example is: country, continent, planet, solar system, galaxy.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• classify the numbers in question 1, part a • explain why 8 belongs to more than one set in question 1, part b.	• present an analogy for the nested pattern of number sets in the real number system in question 2.

General and Specific Outcomes

General Outcome

Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.

Specific Outcome

2.4 Use approximate representations of irrational numbers. [R, T]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and	[T]	Technology
	Mental Mathematics	[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 5.2.
- Both mental estimates and calculator approximations are appropriate for this specific outcome.
- This specific outcome can be addressed throughout the curriculum. Teachers may choose to address this outcome within the topics of trigonometry or measurement.
- Estimates of irrational numbers should be restricted to only square roots of whole numbers between 1 and 144.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• use a calculator to find approximations of irrational numbers • estimate values of square roots.	• evaluate the precision of different approximations of an irrational number.

General and Specific Outcomes

General Outcome

Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.

Specific Outcome

2.4   Use approximate representations of irrational numbers. [R, T]

[   ]   Conceptual
[   ]   Procedural
[✓]   Problem-solving

Question:

1.   Compare the results of using different approximations for $\sqrt{2}$.

    a.   Calculate $\sqrt{2} \times \sqrt{2}$ as the approximation 1.4 × 1.4.

    b.   Calculate $\sqrt{2} \times \sqrt{2}$ using a calculator.

    c.   Which is more precise? Why?

Solution:

1.   a.   1.4 × 1.4 = 1.96

    b. $\sqrt{2} \times \sqrt{2}$ = 2.00

    c.   The calculator answer in part b is more precise, as the calculator uses a 10-decimal place approximation for $\sqrt{2}$, rather than the 1-decimal place approximation used in part a.

Question:

2.   Use approximation to explain why $\sqrt{35} + \sqrt{67}$ is not equal to $\sqrt{102}$.

Solution:

2. $\sqrt{35}$ is approximately 6, and $\sqrt{67}$ is approximately 8; so $\sqrt{35} + \sqrt{67}$ is approximately 14, while $\sqrt{102}$ is approximately 10.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• complete parts a and b in question 1 • explain why $\sqrt{35} + \sqrt{67} \neq \sqrt{102}$ in question 2.	• complete part c in question 1.

General and Specific Outcomes

General Outcome

Use basic arithmetic operations on real numbers to solve problems.

Specific Outcome

2.5 Communicate a set of instructions used to solve an arithmetic problem including instructions that follow the algebraic format of the formula. [C]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and	[T]	Technology
	Mental Mathematics	[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 5.3.
- Students should be able to communicate keying instructions that follow the algebraic format of the formula.
- Students should be made aware that the formula may need reformatting, **using parentheses**, before the keying sequence will work.
- Students should gain experience in reformatting formulas so that they work successfully on a particular calculator or spreadsheet program.
- Keying sequences should be covered in the appropriate problem contexts.
- This specific outcome can be addressed on an "as needed" basis when addressing other curriculum outcomes.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• create a set of keystrokes but cannot explain the reason for the order.	• explain the reason for the order of keystrokes.

General and Specific Outcomes

General Outcome

Use basic arithmetic operations on real numbers to solve problems.

Specific Outcome

2.5 Communicate a set of instructions used to solve an arithmetic problem including instructions that follow the algebraic format of the formula. [C]

[   ]   Conceptual
[✓]   Procedural
[   ]   Problem-solving

Question:

1. a. Given the formula for the surface area of a cylinder,
        $SA = 2\pi r(r + h)$, write a set of instructions that will allow another student to use the formula when the radius is 5 cm and the height is 12 cm.
    b. Create a set of keystroke instructions for using technology to find the volume of a sphere with a radius of 5 cm.

Solution:

1. a. • add the radius and the height
        • multiply the result by the radius
        • multiply the result by 2π

    or

    $2 \times 2\text{nd } \pi \times 5 \times (5 + 12) \text{ ENTER}$
             ⇑   ⇑
            may   or =
            be   on many
            omitted calculators

    b. Standard formula:
        $V = \frac{4}{3}\pi r^3$

    $4 \times 2\text{nd } \pi \times 5 \wedge 3 \div 3 \text{ ENTER}$
                        ⇑
                       or =
                     on many
                     calculators

    or

    $5 \wedge 3 \times 4 \times 2\text{nd } \pi \div 3 \text{ ENTER}$
                            ⇑
                          or =
                        on many
                        calculators

(continued)

General and Specific Outcomes

General Outcome

Use basic arithmetic operations on real numbers to solve problems.

Specific Outcome

2.5 Communicate a set of instructions used to solve an arithmetic problem including instructions that follow the algebraic format of the formula. [C]

[ ] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)
Question:

2. Create a set of keystrokes that will evaluate A, where

$$\cos A = \left[\frac{2^2 + 3^2 - 2.5^2}{2(2)(3)} \right].$$

Solution:

2. One of many solutions:

2nd cos $((2 \wedge 2 + 3 \wedge 2 - 2.5 \wedge 2) \div (2*2*3))$ ENTER
↑
or =
on many
calculators

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• write a set of instructions or a set of keystrokes that will solve formulas similar to those in question 1.	• rewrite formulas, such as the one in question 2, into a form that is compatible with the calculator being used • write the set of keystrokes that allow a user to calculate a value for A in formulas such as the one in question 2.

General and Specific Outcomes

General Outcome

Use basic arithmetic operations on real numbers to solve problems.

Specific Outcome

2.6 Perform arithmetic operations on irrational numbers, using appropriate decimal approximations. [E, T]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and	[T]	Technology
	Mental Mathematics	[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 5.4.
- For this specific outcome, precise calculations are done on the calculator, while estimates can be done mentally.
- Computations using exact values are not included in applied mathematics. They are part of Pure Mathematics 10, specific outcome 5.6.
- This specific outcome can be addressed throughout the curriculum.
- Students need to be reminded that decimal values should not be rounded until the end of a series of calculations.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- perform the operations, using a calculator - manipulate equations involving simple operations of multiplying and dividing.	- manipulate equations prior to performing operations - manipulate equations that involve roots, powers or order of operations.

General and Specific Outcomes

General Outcome

Use basic arithmetic operations on real numbers to solve problems.
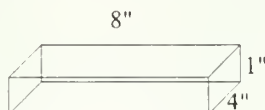
Specific Outcome

2.6 Perform arithmetic operations on irrational numbers, using appropriate decimal approximations. [E, T]

Sample Task

[   ] Conceptual
[✓] Procedural
[   ] Problem-solving

Question:

1. a. How many cm^3 of wax are needed to make a spherical candle with a radius of 5 cm?
    b. What would be the radius of a spherical candle made from the block of wax in the diagram below?

Solution:

1. a. $V = \frac{4}{3}\pi(5)^3$
        $= 523.6 \text{ cm}^3$

    b. Volume of wax $= lwh$
        $= 8 \text{ in} \times 4 \text{ in} \times 1 \text{ in}$
        $= 32 \text{ in}^3$

        Radius of candle:
        $V = \frac{4}{3}\pi r^3$
        $3V = 4\pi r^3$
        $\frac{3V}{4\pi} = r^3$
        $r = \sqrt[3]{\frac{3V}{4\pi}}$
        $= \sqrt[3]{\frac{3(32)}{4\pi}}$
        $= \sqrt[3]{7.639 \text{ in}^3}$
        $= 1.97 \text{ in}$

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• determine the volume of wax in question 1, part a • calculate the volume of wax in question 1, part b.	• calculate the radius of the candle in question 1, part b.

[C] Communication [PS] Problem Solving
[CN] Connections [R] Reasoning
[E] Estimation and [T] Technology
Mental Mathematics [V] Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 1.5.
- Table formats can be given to students and modifications made by editing table entries.
- At the standard of excellence, table formats are not given.
- Students can be expected to insert rows and columns as part of the modifications.
- Tables used in this specific outcome should be relatively simple and limited to 24 cells.
- The ability of students to work with spreadsheets may depend on their background from junior high school and from career and technology studies courses.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• modify given table formats • insert rows and columns to accommodate changed contexts • calculate the amounts for the cells in a recursive table.	• devise a table format to solve a problem • create and modify recursive tables.

General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

2.7 Create and modify tables from both recursive and nonrecursive situations. [PS, T, V]

[   ] Conceptual
[   ] Procedural
[✓] Problem-solving

Question:

1. A 15-year-old student wants to save $2000 to buy a car when he turns 18. He is able to save $50 a month from his part-time job. In the first year the interest on the investment account is 8% of the balance at the end of the year. For the second year it is 7%, and for the third year it is 8.5%.

   a. Create a table to determine if the student will have enough money to buy a $2000 car after 3 years. The headings for the columns in your table should be:

Column	Heading
A	Year
B	Starting Balance
C	Yearly Savings
D	Starting Balance + Yearly Savings
E	Interest Rate (%)
F	Amount of Interest
G	Ending Balance

   b. Explain how to calculate the amounts for the cells in columns **D**, **F** and **G**.

Solution:

1. a.

A	B	C	D	E	F	G
Year	Starting Balance	Yearly Savings	Starting Balance + Yearly Savings	Interest Rate (%)	Amount of Interest	Ending Balance
1	$0.00	$600.00	$600.00	8	$48.00	$648.00
2	$648.00	$600.00	$1248.00	7	$87.36	$1335.36
3	$1335.36	$600.00	$1935.36	8.5	$164.51	$2099.87

The student will be able to afford the car.

b. Column D: take the sum of columns B and C for each row.

    Column F: multiply the value in column D with the interest rate, expressed as a decimal, in column E for each row.

    Column G: take the sum of columns D and E for each row.

(continued)

General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

2.7 Create and modify tables from both recursive and nonrecursive situations. [PS, T, V]

Sample Tasks

[   ]   Conceptual
[   ]   Procedural
[✓]   Problem-solving

(continued)
Question:

2.   Use the table to answer the questions.

Year	Opening Balance	Interest Rate (%)	Interest Earned	Closing Balance
1	$2500.00	6	$150.00	$2650.00
2	$2650.00	6	$159.00	$2809.00
3	$2809.00	6	$168.54	$2977.54
4	$2977.54	6	$178.65	$3156.19

a.   Calculate the total interest earned during the four years.
b.   Suppose the interest rate increased to 9% in Year 4. What would the closing balance be at the end of that year?
c.   Let A dollars represent the initial investment. Write expressions for the interest earned and the closing balance in Year 1.

Solution:

2.   a.   Total interest earned is either $(150 + 159 + 168.54 + 178.65)$ or $(3156.19 - 2500)$. In either case, interest earned is $656.19.
     b.   The closing balance would be $3245.52.
     c. $IE = 0.06A$ for interest earned.
          $CB = 1.06A$ or $CB = A + 0.06A$ for closing balance.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• create the table in question 1, part a, with guidance • explain how to calculate the amounts for most of the cells in question 1, part b • answer question 2, parts a and b.	• create the table in question 1, part a, independently • explain how to calculate the correct amounts for all of the cells in question 1, part b • correctly answer question 2, part c.

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(August 2002)

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General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

2.8 Use and modify a spreadsheet template to model recursive situations. [PS, T, V]

[C] Communication [PS] Problem Solving
[CN] Connections [R] Reasoning
[E] Estimation and [T] Technology
Mental Mathematics [V] Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 1.6.
- Generally, spreadsheet formats are given to students and modifications can be made by editing table entries and formulas.
- At the standard of excellence, spreadsheet formats are not given, and students can be expected to insert/delete rows and columns as part of the modifications.
- The ability of students to work with spreadsheets may depend on their background from junior high school and from career and technology studies courses.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- insert or delete rows and columns as needed - apply spreadsheet formulas to determine cell entries - modify provided formulas in spreadsheet templates.	state or create new formulas in modifying spreadsheets.

General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

2.8 Use and modify a spreadsheet template to model recursive situations. [PS, T, V]

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. A young couple is taking out a loan for some home renovations. The principal amount of the loan is $20 000, and they are given an interest rate of 5%. The loan is for five years and the bank informs the couple that their annual payments will be $4619.50 to pay off the loan. The bank gave the couple the following table.

	A	B	C	D	E	F
		Opening	Interest	Interest	Annual	Closing
1	Year	Balance	Rate(%)	Charged	Payment	Balance
2	1	$20 000.00	5	$1000.00	$4619.50	$16 380.50
3	2	$16 380.50	5	$819.03	$4619.50	$12 580.03
4	3	$12 580.03	5	$629.00	$4619.50	$8 589.53
5	4	$8 589.53	5	$429.48	$4619.50	$4 399.51
6	5	$4 399.51	5	$219.98	$4619.50	$0.00

a. The couple budgeted to make annual payments of $3000 and planned to make a lump sum payment for the remainder of the loan at the end of five years. Modify the template provided to calculate the lump sum payment and the total interest paid in the five years.

b. The couple find they cannot afford to make the lump sum payment after five years. How long will it take to pay off the original loan using $3000 for an annual payment?

c. Unexpectedly, the couple receives gifts and bonuses totalling $1000 in years 3 and 5. They decide to apply this against their loan. Modify the table in part b and determine how much sooner they will pay off the loan.

Solution:

1. a.

	A	B	C	D	E	F
		Opening	Interest	Interest	Annual	Closing
1	Year	Balance	Rate(%)	Charged	Payment	Balance
2	1	$20 000.00	5	$1000.00	$3000.00	$18 000.00
3	2	$18 000.00	5	$900.00	$3000.00	$15 900.00
4	3	$15 900.00	5	$795.00	$3000.00	$13 695.00
5	4	$13 695.00	5	$684.75	$3000.00	$11 379.75
6	5	$11 379.75	5	$568.99	$3000.00	$8 948.74

The lump sum payment at the end of five years is $8948.74, and the total interest is $3948.74.

(continued)

General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

2.8 Use and modify a spreadsheet template to model recursive situations. [PS, T, V]

[ ] Conceptual
[ ] Procedural
[✓] Problem-solving

(continued)
Solution:

1. b.

	A	B	C	D	E	F
		Opening	Interest	Interest	Annual	Closing
1	Year	Balance	Rate %	Charged	Payment	Balance
2	1	$20 000.00	5	$1000.00	$3000.00	$18 000.00
3	2	$18 000.00	5	$900.00	$3000.00	$15 900.00
4	3	$15 900.00	5	$795.00	$3000.00	$13 695.00
5	4	$13 695.00	5	$684.75	$3000.00	$11 379.75
6	5	$11 379.75	5	$568.99	$3000.00	$8 948.74
7	6	$8 948.74	5	$447.44	$3000.00	$6 396.17
8	7	$6 396.17	5	$319.81	$3000.00	$3 715.98
9	8	$3 715.98	5	$185.80	$3000.00	$901.78
10	9	$901.78	5	$45.09	$946.87	$0.00

It will take the couple nine years to pay off the loan using payments of $3 000 per year. In the ninth year the payment is only $946.87.

1. c.

	A	B	C	D	E	F
		Opening	Interest Rate	Interest	Annual	Closing
1	Year	Balance	(%)	Charged	Payment	Balance
2	1	$20 000.00	5	$1000.00	$3000.00	$18 000.00
3	2	$18 000.00	5	$900.00	$3000.00	$15 900.00
4	3	$15 900.00	5	$795.00	$4000.00	$12 695.00
5	4	$12 695.00	5	$634.75	$3000.00	$10 329.75
6	5	$10 329.75	5	$516.49	$4000.00	$6 846.24
7	6	$6 846.24	5	$342.31	$3000.00	$4 188.55
8	7	$4 188.55	5	$209.43	$3000.00	$1 397.98
9	8	$1 397.98	5	$69.90	$1467.88	$0.00
10	9	$–	5	$–	$ –	$ -

The loan would be paid off 1 year sooner.

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• edit the table for the $3000.00 annual payment • interpret the closing balance as the required lump sum payment in question 1, part a • find the total interest paid over the five years in question 1, part a • extend the table, and identify when the closing balance is zero in question 1, part b.	• make any modifications required for question 1, part c.

General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

2.9 Solve problems involving combinations of tables, using:
- addition or subtraction of two tables
- multiplication of a table by a real number
- algebraic processes to build spreadsheet functions and templates.

[PS, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Students can be expected to use the basic principles of spreadsheet operations.
- More advanced programming techniques, such as absolute and relative references, or pasted-in library functions, can be useful here. However, they are not required for this specific outcome.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• combine spreadsheets and tables when provided with formulas.	• combine spreadsheets and tables when formulas are not given.

General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

2.9 Solve problems involving combinations of tables, using:
- addition or subtraction of two tables
- multiplication of a table by a real number
- algebraic processes to build spreadsheet functions and templates.

  [PS, T, V]

Question:

1. A farmer takes out two loans with terms as shown in the following tables.

Farm Machinery Loan

	A	B	C	D	E	F
1	Year	Opening Balance	Interest Rate (%)	Interest Paid	Yearly Payment	Closing Balance
2	1	$100 000.00	6.0	$6000.00	$15 000.00	$91 000.00
3	2	$ 91 000.00	6.0	$5460.00	$15 000.00	$81 460.00
4	3	$ 81 460.00	6.0	$4887.60	$15 000.00	$71 347.60

Livestock Loan

	A	B	C	D	E	F
1	Year	Opening Balance	Interest Rate (%)	Interest Paid	Yearly Payment	Closing Balance
2	1	$200 000.00	9.0	$18 000.00	$45 000.00	$173 000.00
3	2	$173 000.00	9.0	$15 570.00	$45 000.00	$143 570.00
4	3	$143 570.00	9.0	$12 921.30	$45 000.00	$111 491.30

a. Combine the two tables into one, with the following headings for the columns.

Combined Loan

	A	B	C	D	E
1	Year	Opening Balance	Interest Paid	Yearly Payment	Closing Balance
2	1				
3	2				
4	3				

b. The bank makes an error, and the yearly payments are reversed. So, $15 000 is applied to the livestock loan, and $45 000 is applied to the farm machinery loan, beginning in the third year. Use tables to predict what will happen.

Solution:

Combined Loan

1. a.

	A	B	C	D	E
1	Year	Opening Balance	Interest Paid	Yearly Payment	Closing Balance
2	1	$300 000.00	$24 000.00	$60 000.00	$264 000.00
3	2	$264 000.00	$21 030.00	$60 000.00	$225 030.00
4	3	$225 030.00	$17 808.90	$60 000.00	$182 838.90

(continued)

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

2.9 Solve problems involving combinations of tables, using:
  • addition or subtraction of two tables
  • multiplication of a table by a real number
  • algebraic processes to build spreadsheet functions and templates.
  [PS, T, V]

Sample Tasks

[   ]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

(continued)
Solution:

1. b.

Farm Machinery Loan

	A	B	C	D	E	F
	Year	Opening Balance	Interest Rate (%)	Interest Paid	Yearly Payment	Closing Balance
1						
2	3	$81 460.00	6.0	$4887.60	$45 000.00	$41 347.60
3	4	$41 347.60	6.0	$2480.86	$45 000.00	$1 171.54

Livestock Loan

	A	B	C	D	E	F
	Year	Opening Balance	Interest Rate (%)	Interest Paid	Yearly Payment	Closing Balance
1						
2	3	$143 570.00	9.0	$12 921.30	$15 000.00	$141 491.30
3	4	$141 491.30	9.0	$12 734.25	$15 000.00	$139 227.55
4	5	$139 227.55	9.0	$12 530.47	$15 000.00	$136 758.02

The farm machinery loan will be more than paid off at the end of the fourth year. The livestock loan, on the other hand, will take a very long time to pay off, as only a small amount of the principal is paid off each year.

Question:

2. To compare the standard of living in different countries, surveys were conducted in Canada and the United States. The results were as follows.

Income and Cost of Living in Canada: (Canadian dollars)

Income	Housing	Food	Transport	Utilities	Taxes
20 000	7 000	4 000	3 000	3 200	2 500
60 000	16 000	6 000	9 000	5 000	14 000
100 000	23 000	7 000	14 000	5 300	37 000

Income and Cost of Living in the United States: (US dollars)

Income	Housing	Food	Transport	Utilities	Taxes
15 000	4 500	3 000	2 000	2 700	1 000
50 000	12 000	4 700	6 500	4 000	9 000
75 000	14 000	5 100	11 000	4 200	20 000

Before a comparison can be made, the same units must be used. Use the exchange rate of $1.00 US = $1.50 Canadian to determine income and the cost of living in the United States in Canadian dollars.

(continued)

General and Specific Outcomes

General Outcome

Describe and apply arithmetic operations on tables to solve problems, using technology as required.

Specific Outcome

2.9 Solve problems involving combinations of tables, using:
- addition or subtraction of two tables
- multiplication of a table by a real number
- algebraic processes to build spreadsheet functions and templates.

[PS, T, V]

Sample Tasks

[   ]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

(continued)
Solution:

2.

Income and Cost of Living in the United States: (Canadian dollars)

Income	Housing	Food	Transport	Utilities	Taxes
22 500	6 750	4 500	3 000	4 050	1 500
75 000	18 000	7 050	9 750	6 000	13 500
112 500	21 000	7 650	16 500	6 300	30 000

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• combine the two tables in question 1 • multiply all table entries by 1.50 to convert to Canadian dollars in question 2.	• predict the outcome in question 1, part b, when the yearly payments are changed.

STANDARDS IN RELATIONS AND FUNCTIONS

GENERAL OUTCOME

- Examine the nature of relations with an emphasis on functions.

SPECIFIC OUTCOMES

3.1 Plot linear and nonlinear data, using appropriate scales. [C, V]

3.2 Represent data, using function models. [CN, PS, V]

3.3 Use a graphing tool to draw the graph of a function from its equation. [C, T, V]

3.4 Describe a function in terms of:
- ordered pairs
- a rule, in word or equation form
- a graph.

[C, CN, V]

3.5 Use function notation to evaluate and represent functions, incorporating contextually appropriate variables; such as volume as a function of time in the form $V = V(t)$ rather than $y = f(x)$. [C, PS]

3.6 Determine the domain and range of a relation from its graph. [PS, V]

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.1 Plot linear and nonlinear data, using appropriate scales. [C, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and	[T]	Technology
	Mental Mathematics	[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 3.1.
- Data collection can be time consuming. If the data are only relevant to one specific outcome, it may be appropriate to provide the data. However, if students collect their own data, many other specific outcomes, notably specific outcomes 1.1 to 1.6, can also be addressed.
- It is important to discuss whether or not it is appropriate to connect points on a graph.
- When graphing manually, students must be able to choose appropriate scales; when using a graphing calculator, students must be able to choose the appropriate window settings.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- plot data using paper and pencil as well as a graphing calculator - choose appropriate scales, axis labels and titles when drawing graphs - choose appropriate window settings when using a graphing calculator.	make predictions and discuss trends on graphs.

General and Specific Outcomes
General Outcome
Examine the nature of relations with an emphasis on functions.
Specific Outcome
3.1   Plot linear and nonlinear data, using appropriate scales. [C, V]

Question:

1.   The mass of a beaker is recorded when the beaker contains varying volumes of ethanol. The results of the experiment are recorded in the table below.

Volume of Ethanol (mL)	Mass of Beaker and Liquid (g)
0	90
50	129
100	168
150	207
200	246

Plot this data on a scatterplot, using appropriate scales, and answer the following questions.

a.   Assuming that this pattern continues, determine the mass of the beaker and liquid when 250 mL of ethanol is present.

b.   When a volume of 200 mL of ethanol is in the beaker, determine the mass of the ethanol alone.

Solution:

1.   **Masses and Volumes of Different Ethanol Samples**
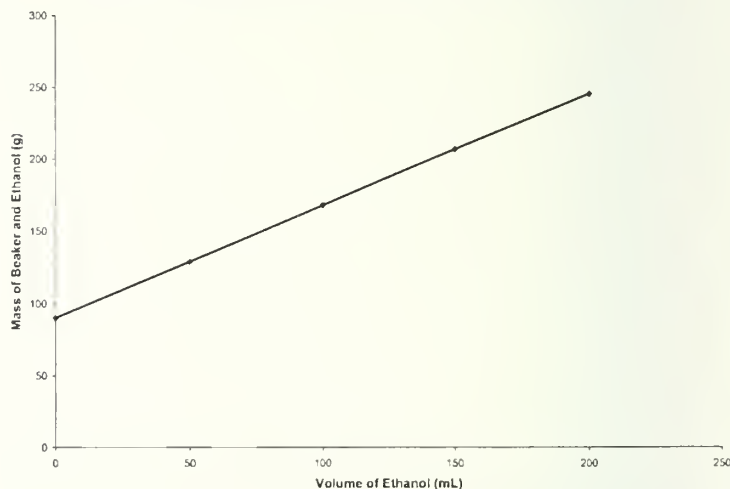
a.   The mass will be about 300 g when 250 mL of ethanol is present.

b.   The mass of the ethanol alone will be (246 – 90) g = 156 g.

(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.1   Plot linear and nonlinear data, using appropriate scales. [C, V]

(continued)
Question:

2.   **Materials:**
     Hot tap water, glass or metal cup, Celsius thermometer, clock or watch
     Task:
     Record, in a table of values, the temperature of a sample of hot tap water as it cools to room temperature. Using appropriate scales, plot your data for time and temperature on a coordinate plane.
     Analysis:
     a.   Is it appropriate to connect the points on the scatterplot? Why or why not?
     b.   Is this a linear function? Explain why you answered as you did.
     c.   Temperature is a measure of the average kinetic energy of the molecules. What will be the trend of the graph after the water is cooled to room temperature?
     d.   If you had used a styrofoam cup for your water sample, explain how your graph might change.

Solution:

2.

Sample Data Table

Time (min)	Temperature (°C)
0	45.0
1	42.5
2	40.3
3	38.2
4	36.4
5	34.8
6	33.3
7	32.0
8	30.8
9	29.7
10	28.7

Cooling of Hot Tap Water
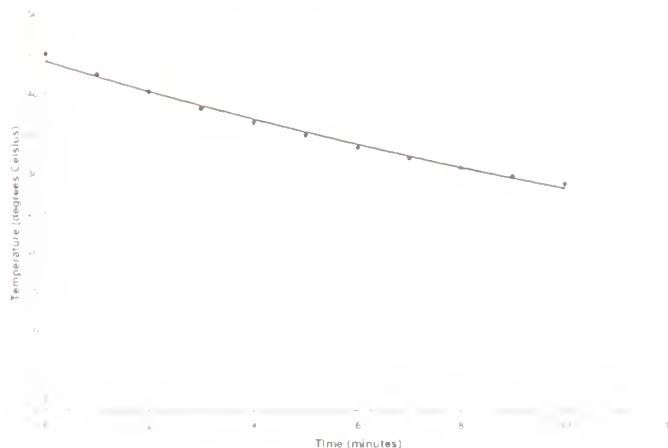
(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.1   Plot linear and nonlinear data, using appropriate scales. [C, V]

*(continued)*
Solution:

2.   a.   It is not appropriate to connect the points. The graph should be a smooth best-fit curve, because time is continuous and there is a corresponding temperature for every time.
     b.   This is a nonlinear function. The water cooled quickly at first and then more slowly as it approached room temperature.
     c.   The temperature will remain constant at about 20°C. The graph approaches a straight horizontal line.
     d.   In a styrofoam cup, the water would take longer to cool, so the graph would be less steep at first.

Question:

3.   **Materials:**
     Balls of the same material in various sizes; overflow containers; measuring cylinder, beaker or cup; string; centimetre ruler; aluminum pan.
     Task:
     Measure and record the circumference of each ball. Fill the overflow container with water to the overflow opening. Place a collecting vessel under the opening. Submerge one ball in the water, collecting the water to determine the volume of the ball. Repeat for the other balls.
     Analysis:
     Plot the data of the relationship between the circumference and volume. Estimate a line or curve of best fit.
     a.   Is this a linear or a nonlinear relationship?
     b.   Predict the value of V when C is zero.

Solution:

3.   **Sample Data Table**

Circumference (cm)	Volume (mL)
9.6	14.9
12.3	31.4
18.9	114.0
21.5	167.8

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.1 Plot linear and nonlinear data, using appropriate scales. [C, V]

(continued)
Solution:

3. **Circumference and Volume of Spheres**
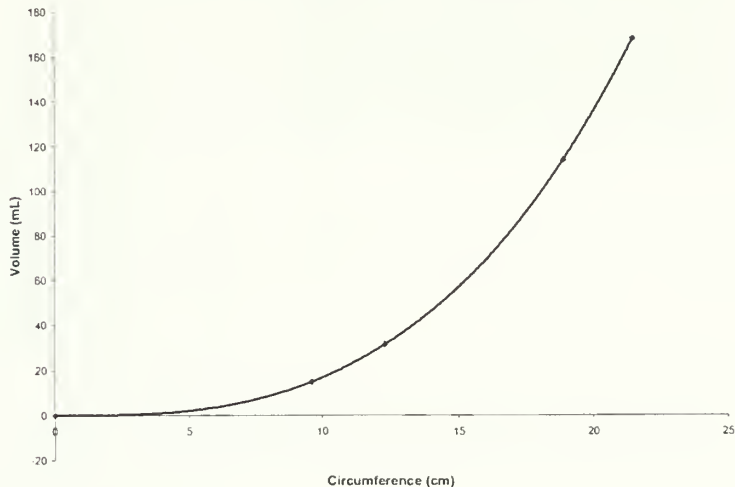
a. This is a nonlinear relationship.
b. When C is zero, V will also be zero.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• generate an appropriate graph for question 1 • extrapolate to find the required mass in question 1, part a • complete the tasks in questions 2 and 3, with assistance • provide answers to parts a through d of question 2 • identify the relationship in question 3, part a, as a nonlinear relationship.	• provide a correct answer to question 1, part b • complete the tasks in questions 2 and 3, without assistance • estimate a smooth curve for question 2, which suggests a nonlinear relationship • provide complete answers, with explanation, to parts a through d of question 2 • predict that the graph will pass through the origin in question 3, part b.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.2 Represent data, using function models. [CN, PS, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 4.1.
- Equations and precise scales for graphs are not absolutely necessary for this specific outcome. However, labels and titles should be clear, and graphs should be in appropriate proportion with scales, if required.
- Students are expected to recognize graphs from scenarios and provide scenarios for graphs.
- It is important for students of applied mathematics to recognize the relationship between graphs and real-life scenarios. This realization will benefit students with other specific outcomes.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• generate sketches of graphs, given real-life relationships between variables • describe a real-life matching situation for graphs of functions, with the descriptions roughly incorporating the meanings of key points.	• describe a real-life matching situation for graphs of functions, including stating the meanings and values of any key points • incorporate quantification into the characteristics and descriptions of graphs.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.2 Represent data, using function models. [CN, PS, V]

[✓] Conceptual
[   ] Procedural
[   ] Problem-solving

Question:

1. Provide a scenario for the following graph. Clearly label the graph to support your story.
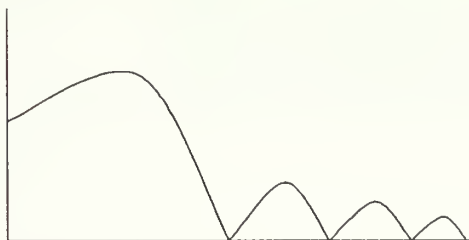
Solution:

1. Throwing a Ball
   A person throws a ball in the air. *A* represents the initial position of the ball. *B* is the maximum height and *C* represents the first time the ball hits the ground. The ball continues to bounce until it comes to rest.
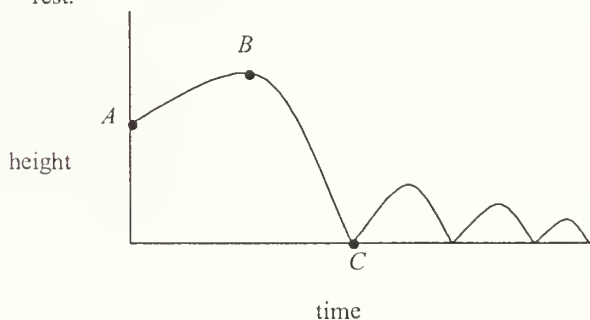
(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.2   Represent data, using function models. [CN, PS, V]

Sample Tasks

[✓]   Conceptual
[   ]   Procedural
[   ]   Problem-solving

(continued)
Question:

2.   Sketch the graph of a woman walking her dog on her lunch hour. Use distance as a function of time. Be creative with your graph. Write the corresponding story for your graph.

Solution:

2.   (One of many possible solutions.)
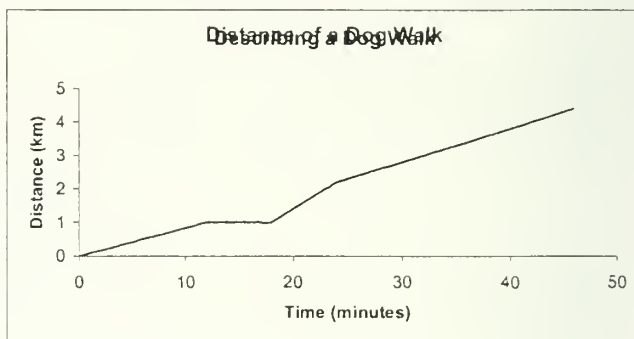
This dog walk shows 12 minutes of walking at a steady, slow pace, covering a distance of 1.0 km. Then there was a resting period, followed by 1.2 km at a faster pace. The return distance of 2.2 km was covered at a moderate pace. The total walking time was 46 minutes.

(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.2 Represent data, using function models. [CN, PS, V]

(continued)
Question:

3. a. Sketch the graph of the number of daylight hours as a function of calendar date for Edmonton, Alberta.
   b. How does the graph change for Sydney, Australia?

Solution:
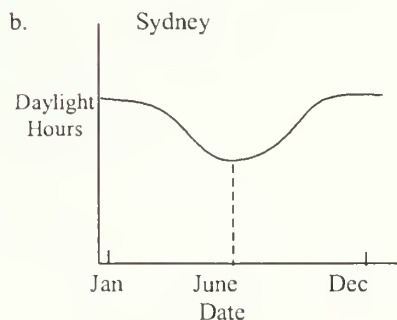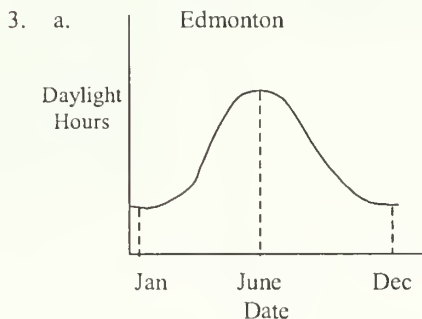
3. a.

Edmonton

b.

Sydney

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• provide a scenario for the graph in question 1, which may miss some key points • label the graph in question 1, but the labelling may be incomplete • sketch the graph in question 2, and write the story in a general way • complete the graph in question 3, part a.	• clearly provide all labels for the graph in question 1, and discuss all key points on the graph • sketch the graph in question 2; write the story; and relate the graph to the story by, for example, labelling the graph • adapt the graph of question 3, part a, for Sydney.

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.3 Use a graphing tool to draw the graph of a function from its equation. [C, T, V]

| [C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- The skill of isolating y is a prerequisite for using a graphing calculator. This skill development needs to be emphasized.
- This specific outcome is common with Pure Mathematics 10 specific outcome 4.2.
- This specific outcome is designed to introduce students to the appropriate use of a graphing tool.
- It is not specified if a graphing calculator or a computer is to be used. Most examples are given in terms of a graphing calculator.
- Students are only expected to manipulate linear equations.
- A thorough discussion of window settings is necessary.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• graph functions, using a graphing tool such as a graphing calculator or a computer graphing program • manipulate window parameters so a graph can be seen • carry out one-step or two-step manipulations of linear equations, so a form of the equation is produced that can be entered into the calculator or graphing program.	• carry out multistep manipulations of linear equations, so a form of the equation is produced that can be entered into the calculator or graphing program.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.3   Use a graphing tool to draw the graph of a function from its equation. [C, T, V]

Sample Tasks

[   ]   Conceptual
[✓]   Procedural
[   ]   Problem-solving

Question:

1.   An object was thrown and followed a path according to the equation $h = -t^2 + 6t + 10$, where h is the height in metres and t is the time in seconds.
    a.   Using the following window settings, sketch the graph: $x_{min} = 0$, $x_{max} = 8$, $y_{min} = 0$, $y_{max} = 20$.
    b.   Trace the path, using the trace function on the graphing calculator, and describe the function at key points.
    c.   Are the window settings appropriate? Explain.

Solution:

1.   a.   **Height of a Thrown Object as a Function of Time**
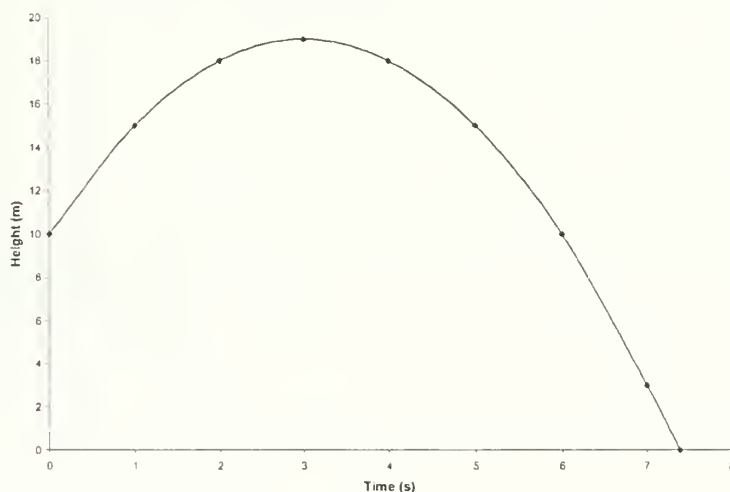
    b.   The graph begins at 10 m, the height from which the object is thrown. It reaches a maximum height of 19 m, then falls to the ground before 8 seconds have passed.
    c.   The window settings are appropriate; x represents the time, which begins at 0 and ends at 8 s, just after the object hits the ground; y represents the height, which never exceeds 20 m.

(continued)

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General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.3 Use a graphing tool to draw the graph of a function from its equation. [C, T, V]

(continued)
Question:

2. Given the equation $y - 2 = x$:
   a. Manipulate the equation so it can be entered into a graphing calculator.
   b. Graph the function. Sketch the graph, providing appropriate window settings.

Solution:

2. a. $y = x + 2$
    b.
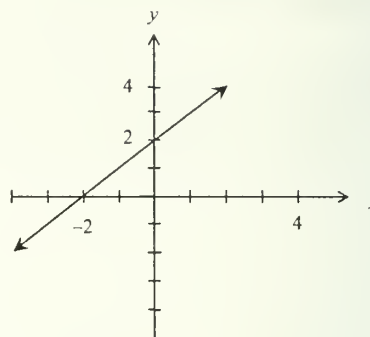
One possible solution may be to have the following window settings: $x_{min} = -4$, $x_{max} = +4$, $y_{min} = -4$, $y_{max} = +4$ (referring to the above sketch).

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• use a graphing calculator to answer question 1, part a • trace the function in question 1, part b • explain the appropriateness of the window settings used in question 1, part c • provide a complete solution to question 2.	• relate key points of the graph to the path of the object in question 1, part b.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.4 Describe a function in terms of:
  • ordered pairs
  • a rule, in word or equation form
  • a graph.
  [C, CN, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

• This specific outcome is common with Pure Mathematics 10 specific outcome 4.3.
• The choice of tests, such as the vertical line test to determine if a relation is a function, is left up to the teacher.
• The use of the table feature on the graphing calculator is an important skill for this specific outcome.
• Relations can be described in terms of input and output values.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:

• make a list of values, list ordered pairs, sketch a graph and write a rule in words for a given function
• determine if a relation is a function, given a:
  – graph and using the vertical line test
  – set of ordered pairs
  – mapping
  – table of values.
	• determine if a relation is a function from the equation
	• write a rule in words for a function whose equation is given
	• explain when a relation is a function.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.4 Describe a function in terms of:
- ordered pairs
- a rule, in word or equation form
- a graph.
[C, CN, V]

Sample Task

[✓] Conceptual
[ ] Procedural
[ ] Problem-solving

Question:

1. The cost of publishing books includes an initial cost of $2000 and a cost of $20 per book thereafter.
   a. Make a table of values for up to 1000 books, using increments of 200.
   b. List all ordered pairs from your table of values.
   c. Graph the relation, using your table of values.
   d. Write an equation that represents this relation.
   e. Does this relation represent a function? Explain.

Solution:

1. a.

no. of books	0	200	400	600	800	1000
cost	$2000	$6000	$10 000	$14 000	$18 000	$22 000

b. (0, 2000), (200, 6000), (400, 10 000), (600, 14 000), (800, 18 000), (1000, 22 000)

c. **Cost of Publishing Books**
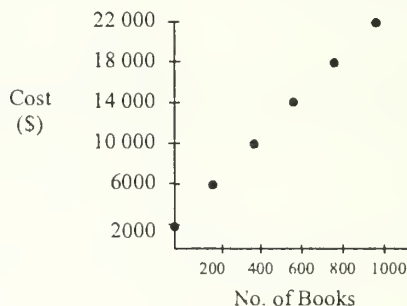
d. $C = 20n + 2\,000$, where C is the total cost and n is the number of books.

e. Yes, it does represent a function. The graph passes the vertical line test; or for every input, there is only one output; or there is a fixed price for any given number of books.

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• complete question 1, parts a, b and c.	• write the equation for this relation in question 1, part d • explain why the relation is a function in question 1, part e.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.5 Use function notation to evaluate and represent functions, incorporating contextually appropriate variables such as volume as a function of time in the form $V = V(t)$ rather than $y = f(x)$. [C, PS]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 4.4.
- Conventional function notation is restricted to rational number inputs.
- Function notation includes spreadsheet formula notation.
- Spreadsheet formula notation can be found in specific outcome 2.9, in the Standards in Number Patterns in Tables section of this document.
- Function notation must use contextually appropriate variables. So volume as a function of time is recorded as $V = V(t)$ not $y = f(x)$.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• evaluate functions for rational number inputs • use a graph or table features to interpret notation.	• explain the significance of the notation.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.5 Use function notation to evaluate and represent functions, incorporating contextually appropriate variables such as volume as a function of time in the form $V = V(t)$ rather than $y = f(x)$. [C, PS]

Sample Task

[   ]   Conceptual
[✓]   Procedural
[   ]   Problem-solving

Question:

1. The cost for hosting a banquet is given by the formula
   $C(n) = 150 + 5n$ where C is the cost and n is the number of people.
   a. Use the graph or the table function to find $C(350)$.
   b. Verify algebraically the value of $C(350)$.
   c. $C(426) = 2280$. Interpret this statement.
   d. If $C(n) = 650$, find n.

Solution:

1. a. $C(350) = 1900$
   b. $C(350) = 150 + 5(350)$
      $\qquad\quad = 150 + 1750$
      $\qquad\quad = 1900$
   c. The cost of hosting a banquet for 426 people is $2280.
   d. Algebraic solution:
      $150 + 5n = 650$
      $\qquad 5n = 500$
      $\qquad\quad n = 100$
      Graphical solution:

Banquet Hosting Costs

Y-axis: Total Banquet cost ($); X-axis: Number of People. Point labelled (100, 650).

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- complete parts a and b - provide a partial solution for parts c and d.	- demonstrate a complete understanding in part c - solve part d algebraically or graphically.

General and Specific Outcomes

General Outcome

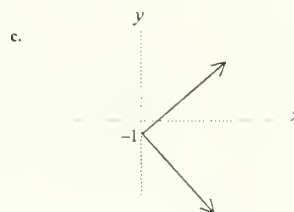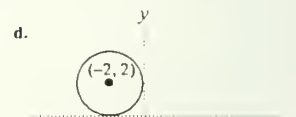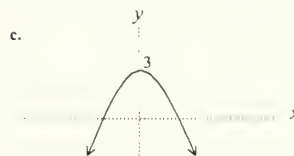Examine the nature of relations with an emphasis on functions.
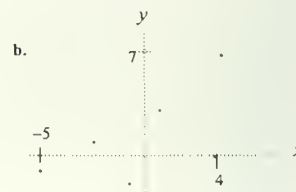
Specific Outcome

3.6 Determine the domain and range of a relation from its graph.
[PS, V]

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 4.5.
- Algebraic derivations for domain and range are not required.
- Graphs must be easily drawn or else must be provided. Students can be expected to use a graphing tool to draw the graph of a function given in function notation form, following specific outcomes 3.3 and 3.5.
- Students are only expected to manipulate linear equations.
- Students are not required to use mathematical notation to express domain and range. A word description is sufficient.
- Number sets, specific outcome 2.3, could be reviewed with this specific outcome.
- The domain and range restrictions are set by the context of the problem.
- Students should be encouraged to relate the concept of domain and range to real-life situations.
- It is appropriate to use a graphing tool for this specific outcome.
- Teachers have the option of showing students how to use the trace function or the calculate function on the graphing calculator for determining range.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- state the domain and range of graphs - interpret the domain and range from word problems, with guidance.	- modify the domain and range of graphs to account for the context of the problem - interpret the domain and range from word problems, without guidance.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.6 Determine the domain and range of a relation from its graph. [PS, V]

Question:

1. Determine the domain and range for each of the graphs.

a.

b.

c.

d.

e.
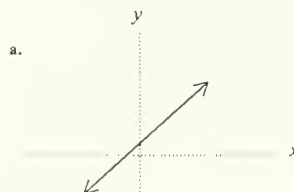
Solution:

1. a.   domain: real numbers
        range: real numbers

    b.   domain: $\{-5, -3, -1, 1, 4\}$
         range: $\{-2, -1, 0, 1, 3, 7\}$

    c.   domain: real numbers
         range: real numbers equal to or less than 3

    d.   domain: real numbers from -4 to 0 inclusive
         range: real numbers from 0 to 4 inclusive

    e.   domain: real numbers greater than or equal to 0
         range: real numbers

(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.6 Determine the domain and range of a relation from its graph.
[PS, V]

(continued)
Question:

2. Determine the domain and range of a circle with centre $(-2, -3)$ and a radius of 3. Draw the graph first.

Solution:

2. Centre $C = (-2, -3)$; $r = 3$
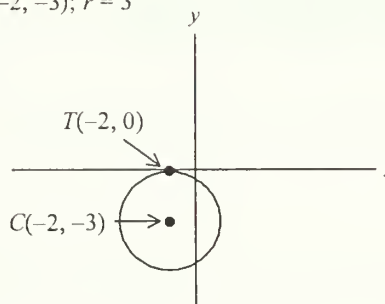
Domain: real numbers from -5 to 1 inclusive
Range: real numbers from -6 to 0 inclusive

Question:

3. a. Sketch the graph of $y = 5000(1.05)^t$.
   b. State the domain and range of the graph obtained in part a.
   c. Suppose the function $A = 5000(1.05)^t$ describes the growth of $5000 invested at 5%, compounded annually. State the domain and range of the function in this context.
   d. Explain why the domain and range are different in parts b and c.

(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

3.6 Determine the domain and range of a relation from its graph.
[PS, V]

Sample Tasks

[✓] Conceptual
[✓] Procedural
[   ] Problem-solving

(continued)
Solution:

3. a.
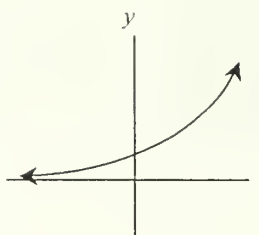
   b. Domain: real numbers
       Range: real numbers greater than 0
   c. Domain: real numbers greater than or equal to 0
       Range: real numbers greater than or equal to 5000
   d. In reality, time is never a negative value. Also the amount of money can never be less than $5000.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• state the domain and range for graphs a, b, c and e in question 1 • state the domain and range for question 2 • sketch the graph in question 3, part a • state the domain and range for question 3, part b.	• state the domain and range for graph d in question 1 • state the domain and range for question 3, part c • explain why the domain and range are different in parts b and c of question 3.

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(August 2002)

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GENERAL OUTCOME

- Solve coordinate geometry problems involving lines and line segments.

SPECIFIC OUTCOMES

4.1 Solve problems involving distances between points in the coordinate plane, including the use of the distance formula $\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ [PS, V]

4.2 Solve problems involving midpoints of line segments, including the use of the midpoint formula $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$. [PS]

4.3 Solve problems involving rise, run and slope of line segments, including the use of $m(\overline{AB}) = \dfrac{y_2 - y_1}{x_2 - x_1}$. [PS, V]

4.4 Solve problems using slopes of:
- parallel lines
- perpendicular lines.
[CN, PS, V]

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.1 Solve problems involving distances between points in the coordinate plane, including the use of the distance formula

$$\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

[PS, V]

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 3.2.
- Coordinates may be written in terms of nonstandard grids; e.g., telephone grid systems and oil drilling diagrams as in Addison-Wesley *Applied Mathematics 10*, Tutorial 5.1.
- Students are not expected to solve linear systems in two variables; problems must be solvable without using systems.
- Linear systems are covered in Applied Mathematics 20.
- It is recommended that teachers review with students the coordinate grid system, ordered pairs and the Pythagorean theorem from Grade 9 Mathematics.
- When using the distance formula, students are expected to show the initial substitution, but it is not necessary to show intermediate steps before the final answer.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• determine distance, using the formula for distance, **OR** plot points on a coordinate plane, and use the Pythagorean theorem to determine distance.	• solve distance problems using a variety of strategies, when intermediate steps are not provided.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.1   Solve problems involving distances between points in the coordinate plane, including the use of the distance formula

$$\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

[PS, V]

[✓]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

Question:

1.   When Boris leaves school, he walks four blocks west and eight blocks north to his home. Alice walks two blocks south and two blocks east to reach her home from the school. If the coordinates of the school are $(0, 0)$, and the blocks are square and identical in size;
     a.   What are the coordinates of each of their homes?
     b.   How far apart are their homes (the shortest aerial distance)? Express the distance as a decimal, to two decimal places.

Solution:

1.   a.   Alice's house is at $(2, -2)$ and Boris' house is at $(-4, 8)$.

1.   b.
$$AB = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$$
$$AB = \sqrt{(-4 - 2)^2 + (8 - -2)^2}$$
$$= \sqrt{(-6)^2 + (10)^2}$$
$$= \sqrt{36 + 100}$$
$$= 11.66 \text{ blocks}$$

OR

Using a grid diagram and counting:
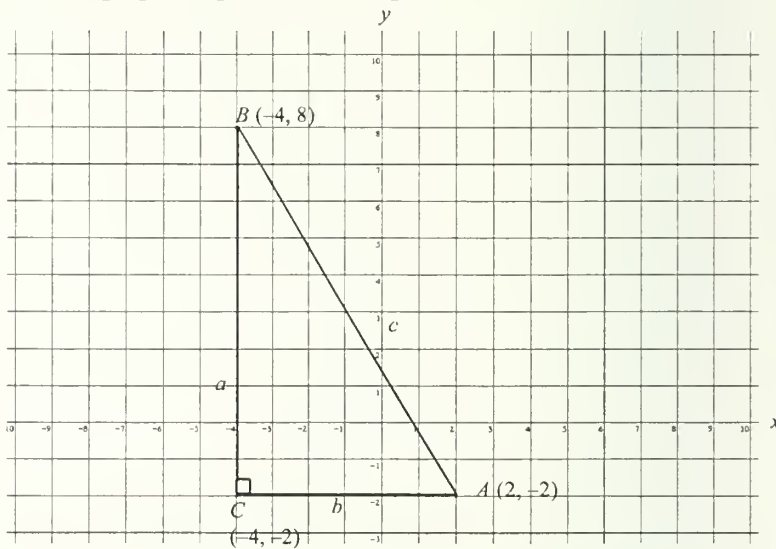
$a = 10$
$b = 6$
Then:
$$c = \sqrt{a^2 + b^2}$$
$$= \sqrt{10^2 + 6^2}$$
$$= \sqrt{136}$$
$$= 11.66 \text{ blocks}$$

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.1   Solve problems involving distances between points in the coordinate plane, including the use of the distance formula

$$\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

[PS, V]

(continued)

Question:

2.   A young couple decided to drive across western Canada in their van. The graph below shows a sketch of their trip as they stopped at several tourist spots.

Distance north of starting point (km)

Distance east of starting point (km)
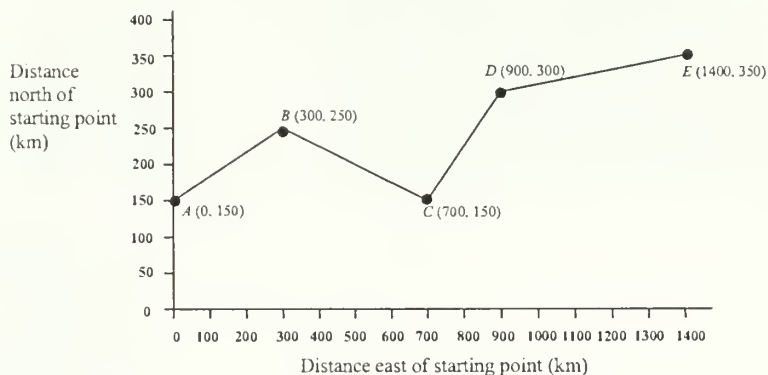
a.   Determine the distance between each of their stops. Show your answers to one decimal place.
b.   Calculate the total distance travelled by the couple.
c.   The van used an average of 1 litre of gasoline for every 12 km travelled, and the average cost of gasoline was 48¢ per litre that summer. How much money did the couple spend on gasoline for their trip?

Solution:

2.   a. $AB = \sqrt{(300 - 0)^2 + (250 - 150)^2} = 316.2$ km

$BC = \sqrt{(700 - 300)^2 + (150 - 250)^2} = 412.3$ km

$CD = \sqrt{(900 - 700)^2 + (300 - 150)^2} = 250.0$ km

$DE = \sqrt{(1400 - 900)^2 + (350 - 300)^2} = 502.5$ km

b.   d = 316.2 km + 412.3 km + 250 km + 502.5 km = 1481 km.

c.   Cost of gasoline = $\dfrac{\$0.48/L}{12 \text{ km/L}} \times 1481$ km = $59.24 .

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.1 Solve problems involving distances between points in the coordinate plane, including the use of the distance formula

$$\overline{AB} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

[PS, V]

Sample Tasks

[✓] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)
Question:

3. A triangular piece of ranch property is listed for sale. It has coordinates of $A(21, 67)$, $B(26, 73)$ and $C(28, 59)$. Assume the coordinates are listed in kilometres.

   a. Determine the perimeter of the property, to the nearest tenth of a kilometre.
   b. A realtor walked around the perimeter of the property at an average speed of 6 km/h. Can you determine the time it took te realtor to walk around the perimeter? Explain.

Solution:

3. a. $AB = \sqrt{(26 - 21)^2 + (73 - 67)^2}$
        $= 7.8$ km
        $BC = \sqrt{(28 - 26)^2 + (59 - 73)^2}$
        $= 14.1$ km
        $CA = \sqrt{(21 - 28)^2 + (67 - 59)^2}$
        $= 10.6$ km
        $P = AB + BC + CA$
        $= 7.8$ km $+ 14.1$ km $+ 10.6$ km
        $= 32.5$ km
    b. Since $speed = \frac{distance}{time}$, you can substitute the known values of distance and speed, then solve for time.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• complete question 1, part a • determine the distance in question 1, part b, if a diagram is given • determine the distance between stops, the total distance travelled and the cost of the gasoline in question 2 • determine the perimeter of the property in question 3, part a, if prompts are provided in the form of intermediate steps.	• determine the distance in question 1, part b, using a coordinate grid or the distance formula • determine the perimeter of the property in question 3, part a, without prompts • provides a complete explanation for question 3, part b.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.2 Solve problems involving midpoints of line segments, including the use of the midpoint formula $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$.

[PS]

[C] Communication   [PS] Problem Solving
[CN] Connections   [R] Reasoning
[E] Estimation and   [T] Technology
     Mental Mathematics   [V] Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 3.3.
- A good way to introduce this topic is to talk about everyday situations related to the equal sharing of the cost of a purchased item; e.g., the cost of a restaurant meal shared between two people.
- Points on line segments should be described in terms of ordered (coordinate) pairs.
- Coordinates may be written in terms of nonstandard grids.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- express midpoints as ordered (coordinate) pairs - solve midpoint problems involving finding a second endpoint, when a diagram is provided.	solve midpoint problems involving finding a second endpoint, when a diagram is not provided.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.2 Solve problems involving midpoints of line segments, including the use of the midpoint formula $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$.
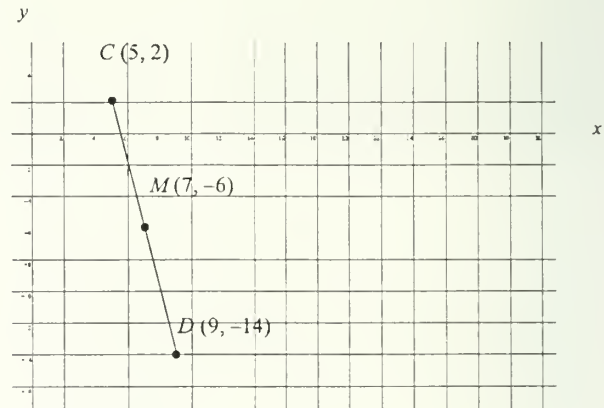
[PS]

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

Question:

1. A grain terminal is to be constructed at the midpoint between two rural centres connected by a railway. When a coordinate grid system is superimposed on a diagram, the coordinates of the centres are (5, 2) and (9, –14). What are the coordinates of the grain terminal?

Solution:

1.

The grain terminal is at (7, –6).
OR

$$M = \left(\frac{5 + 9}{2}, \frac{(2 + -14)}{2} \right)$$

$= (7, -6)$; these are the coordinates of the grain terminal.

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.2 Solve problems involving midpoints of line segments, including the use of the midpoint formula $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2} \right)$.

[PS]

Sample Tasks

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)
Question:

2. Albertville, Browntown and Carsville are on a straight highway as shown in the following diagram.
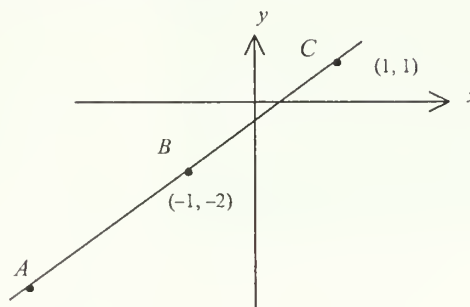
If Browntown is midway between Albertville and Carsville, what are the coordinates of Albertville? Explain how you determined the coordinates.

Solution:

2.   *B* is 2 left and 3 down from *C*;
     A is twice as far, so is 4 left and 6 down from *C*;
     So the coordinates for Albertville are (–3, –5).

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• plot the points in question 1 on a coordinate plane, and determine *M* graphically • find the coordinates of the unknown endpoint in question 2, given the diagram with the points plotted.	• determine the midpoint in question 1 algebraically • find the coordinates of the unknown endpoint in question 2, with the other coordinates provided but no diagram.

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Line Segments / 95
(August 2002)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.3 Solve problems involving rise, run and slope of line segments, including the use of

$$m(\overline{AB}) = \frac{y_2 - y_1}{x_2 - x_1} \, .$$

[PS, V]

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 3.4.
- Coordinates may be written in terms of nonstandard grids.
- For alternative approaches to the concept of slope, refer to Addison-Wesley *Applied Mathematics 10*, Tutorials 6.3 and 6.4.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- find the slope, given the graph of a line - find the slope, given any two points, using the slope formula - distinguish between negative and positive slopes - graph a line, given a point and the slope of the line - identify that vertical lines have an undefined slope and horizontal lines have a slope equal to zero.	- make connections between trigonometry and line segments - explain why vertical lines have an undefined slope and horizontal lines have a slope equal to zero - interpret slope in terms of rate or steepness - interpret the units for slope - interpret the meaning of slope on nonstandard grids.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.3   Solve problems involving rise, run and slope of line segments, including the use of

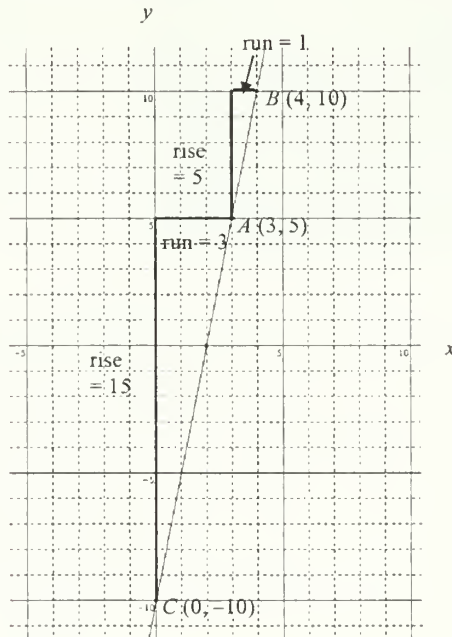$$m(\overline{AB}) = \frac{y_2 - y_1}{x_2 - x_1} \, .$$

[PS, V]

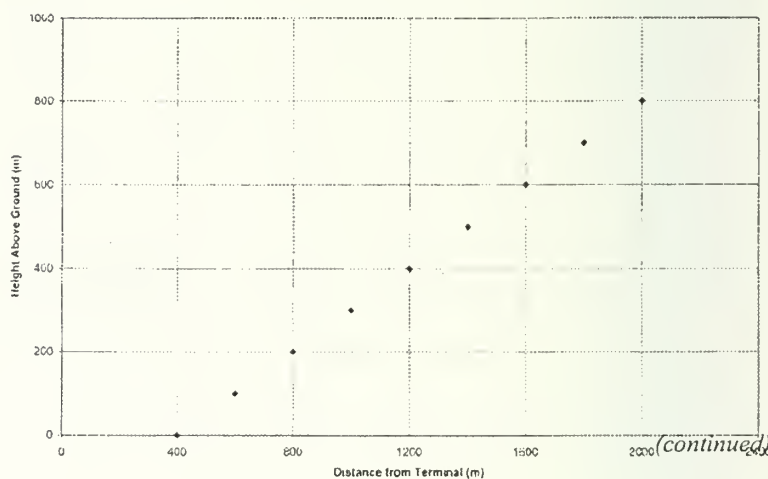[   ]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

Question:

1.   Given $A(3, 5)$ is on a line of slope 5, find another point on the line and label it B on a graphical sketch.

Solution:

1.

Go up 5 and over 1.
Use rise over run to find B graphically.
Another point could be $(4, 10)$ or $(5, 15)$, among others.

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.3 Solve problems involving rise, run and slope of line segments, including the use of

$$m(\overline{AB}) = \frac{y_2 - y_1}{x_2 - x_1}.$$

[PS, V]

Sample Tasks

[   ]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

*Question:*

2. A jet lifts off the runway 400 m from the terminal (0,0) at a slope of $\frac{1}{2}$. Will the jet clear a 700 m tower located 2000 m from the terminal? Explain.

Solution:

2. Use rise over run to trace the slope graphically. The slope of $\frac{1}{2}$ means for every 200 m horizontal motion, the jet climbs 100 m. The dots on the graph represent the jet's position every 200 m. If the jet moves 1600 m horizontally, it moves 800 m vertically. It will clear the tower with 100 m to spare.

Aircraft Height as a Function of Distance from Terminal

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.3 Solve problems involving rise, run and slope of line segments, including the use of

$$m(\overline{AB}) = \frac{y_2 - y_1}{x_2 - x_1}.$$

[PS, V]

[   ]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

(continued)

Question:

3. A computer screen has the following display. It does not conform to a standard coordinate grid.

Origin ────────────────→ 1000
                                    Horizontal
750
Vertical
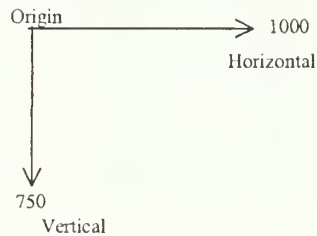
A is (420, 210) and *B* is (800, 530). Plot *A* and *B*, and calculate the slope. Explain why the slope is positive.

Solution:

3.

        A (420, 210)
        •

        B (800, 530)
         •
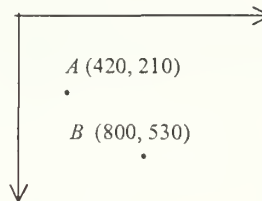
$$\text{Slope} = \frac{530 - 210}{800 - 420}$$

$$= \frac{320}{380} = \frac{16}{19} \text{ or } 0.842$$

The slope is positive because the *x* values and the *y* values are increasing.

(continued)

[ ]	Conceptual
[✓]	Procedural
[✓]	Problem-solving

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.3 Solve problems involving rise, run and slope of line segments, including the use of

$$m(\overline{AB}) = \frac{y_2 - y_1}{x_2 - x_1}.$$

[PS, V]

(continued)

Question:

4. A ladder of length 8.0 m is placed against a vertical wall. For safety reasons, the angle between the ladder and the ground must lie in the range of 75° to 80°. Complete the table below. Describe the change in slope as the angle between the ladder and the ground changes.
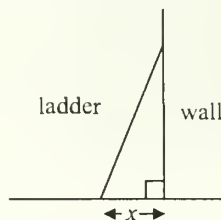
angle	rise (m)	run (m)	slope
75°			
76°			
77°			
78°			
79°			
80°			

Solution:

4.

angle	rise (m)	run (m)	slope
75°	7.73	2.07	3.73
76°	7.76	1.94	4.01
77°	7.79	1.80	4.33
78°	7.83	1.66	4.70
79°	7.85	1.53	5.14
80°	7.88	1.39	5.67

The slope increases as the angle increases.

Note: This question requires knowledge of basic trigonometry and coordinate geometry.

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.3   Solve problems involving rise, run and slope of line segments, including the use of

$$m(\overline{AB}) = \frac{y_2 - y_1}{x_2 - x_1} .$$

[PS, V]

(continued)

Question:

5.   The mass of a beaker is recorded when the beaker contains varying volumes of ethanol. The results of the experiment are recorded in the table below.

Volume of Ethanol (mL)	Mass of Beaker and Liquid (g)
0	90
50	129
100	168
150	207
200	246

Plot this data on a scatterplot, using appropriate scales, and answer the following questions.

a.   Assuming that this pattern continues, determine the mass of the beaker and liquid when 250 mL of ethanol is present.

b.   When a volume of 200 mL of ethanol is in the beaker, determine the mass of the ethanol alone.

c.   The density of a liquid is defined as the mass of 1 mL of the liquid. Determine the density of the ethanol.

d.   What does the slope of the line represent?

(continued)

[	]	Conceptual
[	]	Procedural
[✓	]	Problem-solving

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.3 Solve problems involving rise, run and slope of line segments, including the use of

$$m(\overline{AB}) = \frac{y_2 - y_1}{x_2 - x_1}.$$

[PS, V]

(continued)

Solution:

5. **Masses and Volumes of Different Ethanol Samples**
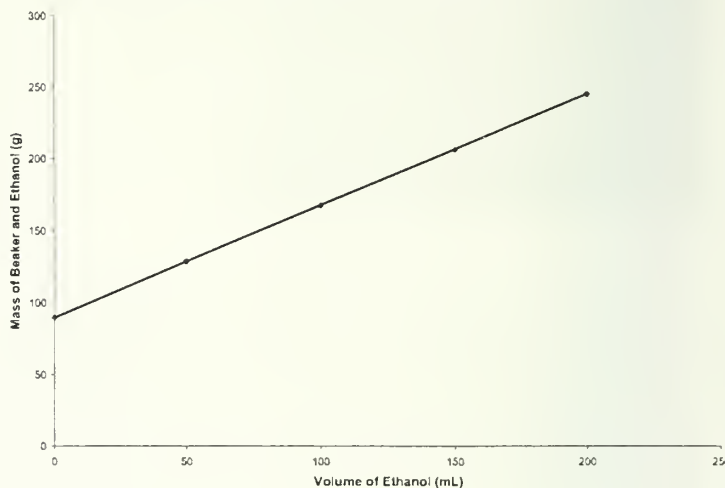
a. The mass will be about 300 g when 250 mL of ethanol is present.
b. The mass of the ethanol alone will be (246 – 90) g = 156 g.
c. Density of ethanol is $\frac{156 \text{ g}}{200 \text{ mL}} = 0.78$ g/mL.
d. The slope of line has the units $\frac{g}{mL}$, so represents the density of ethanol.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- plot point *A*, for question 1, on a coordinate plane, and then use $\frac{\text{rise}}{\text{run}}$ to find a second point - solve question 2 completely - plot the points, and compute a numerical value for the slope in question 3 - solves question 5, parts a and b.	- explain unusual values for slope in terms of the orientation of the grid in question 3 - complete the table of rise, run and slope; and make interpretations from the table in question 4, if a diagram is supplied - solve question 5, parts c and d.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.4 Solve problems using slopes of:
- parallel lines
- perpendicular lines.
[CN, PS, V]

[C] Communication   [PS] Problem Solving
[CN] Connections   [R] Reasoning
[E] Estimation and   [T] Technology
Mental Mathematics   [V] Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 3.6.
- A geometry software program can help students to visualize the relationship between slope and parallel and perpendicular lines, but is not necessary.

| Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• find the slope of a given line • determine if lines are parallel or perpendicular based on their slopes.	• apply knowledge of the slopes of parallel and perpendicular lines to solve problems.

Sample Tasks

[   ]   Conceptual
[✓]   Procedural
[   ]   Problem-solving

Question:

1.   A triangle has coordinates $A(4, 7)$, $B(1, 11)$ and $C(12, 13)$. Verify that $\triangle ABC$ has a right angle at A, by using:
  a.   the slopes of AB and AC
  b.   the Pythagorean theorem.

Solution:

1.   a.
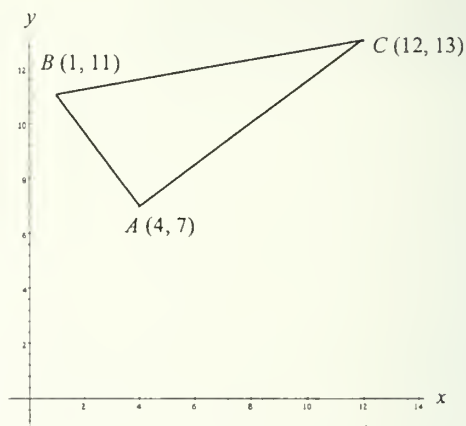
$$\text{slope } AB = \frac{7-11}{4-1}$$
$$= -\frac{4}{3}$$
$$\text{slope } AC = \frac{13-7}{12-4}$$
$$= \frac{3}{4}$$

These slopes satisfy $m_1 m_2 = -1$.

So $AB \perp AC$.

b. $AB^2 = (4-1)^2 + (7-11)^2 = 25$
$AC^2 = (12-4)^2 + (13-7)^2 = 100$
$BC^2 = (13-11)^2 + (12-1)^2 = 125$
So $BC^2 = AB^2 + AC^2$ and $A = 90°$.

(continued)

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.4 Solve problems using slopes of:
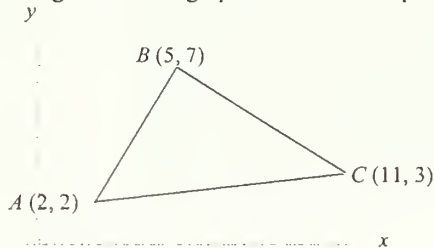- parallel lines
- perpendicular lines.
[CN, PS, V]

Sample Tasks

[   ]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

(continued)
Question:

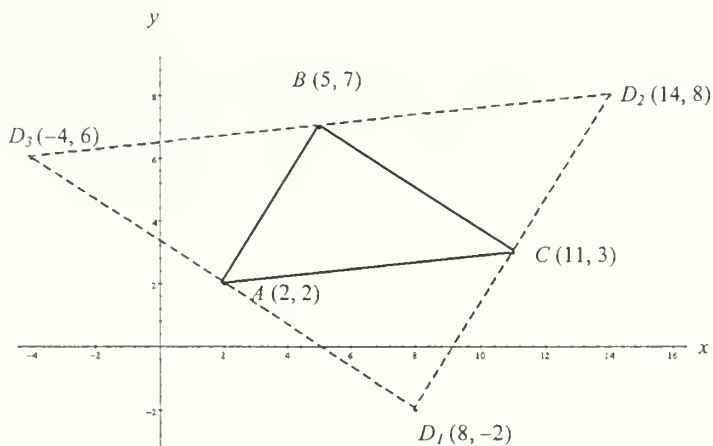2.   Use the following sketch of a graph to answer the question.

With A, B, C as indicated, find a fourth point of a parallelogram. Justify your answer. There may be more than one correct answer.

Solution:

2. $(8, -2)$ for D_1
                                or $(14, 8)$ for D_2
                                or $(-4, 6)$ for D_3

(continued)

[   ]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.4   Solve problems using slopes of:
- parallel lines
- perpendicular lines.
[CN, PS, V]

(continued)
Solution:

2.   Justification

     For D_1 and D_3, AD is parallel to BC and equal in length to BC.

     BC has a rise of –4 and a run of 6. So D_1 is 6 right and 4 down from A, or (8, –2), and D_3 is 6 left and 4 up from A, or (–4, 6).

     BD_2 is parallel to AC and equal in length to AC.

     AC has a rise of 1 and a run of 9. So D_2 is 9 right and 1 up from B, or (14, 8).

Question:

3.   Kia left Edmonton at 9:00 a.m. to drive to Calgary, a distance of 300 km. Edwin also drove from Edmonton to Calgary that day but left at 1 p.m. Each travelled at an average speed of 100 km/h. Graph their trips using time of day on the horizontal axis and distance on the vertical axis.
     a.   Show that the graphs are parallel lines.
     b.   Explain how the relationships between the graphs would change if Edwin's average speed had been 105 km/h instead of 100 km/h.

Solution:

3.   **Tracking Drivers**
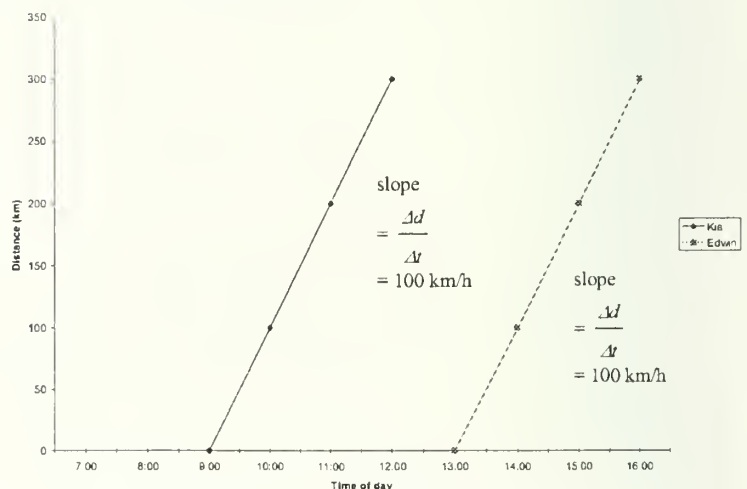
     a.   The slopes are equal, therefore the lines are parallel.
     b.   The lines would no longer be parallel because the slopes would be 100 km/h and 105 km/h.

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.4   Solve problems using slopes of:
   • parallel lines
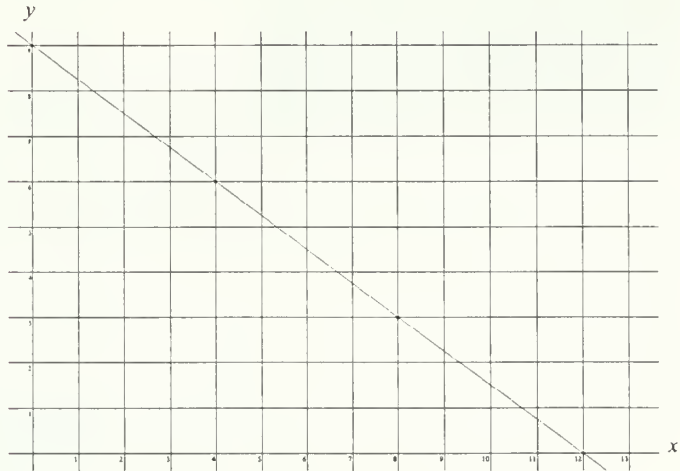   • perpendicular lines.
   [CN, PS, V]

(continued)
Question:

4.   The following diagram shows a city grid of 117 blocks with a main road running across it.

Two intersections with traffic lights are at coordinates (4, 6) and (8, 3). Draw new roads perpendicular to the main road at these lights.
a.   Where do the new roads intersect with the axes?
b.   Show that the two new roads are parallel to one another.
c.   Compare the slope of the main road with the slopes of the new roads.

(continued)

General and Specific Outcomes

General Outcome

Solve coordinate geometry problems involving lines and line segments.

Specific Outcome

4.4   Solve problems using slopes of:
  • parallel lines
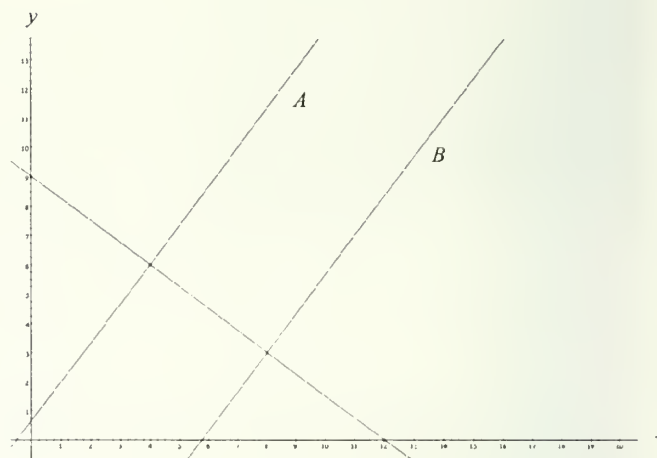 • perpendicular lines.
  [CN, PS, V]

(continued)
Solution:

4.

a.   Road *A* intersects the *y*-axis at (0, 0.6).
     Road *B* intersects the *x*-axis at (5.8, 0).

b.   Slope A = Slope B = $\dfrac{4}{3}$. Therefore roads *A* and *B* are parallel.

c.   Slope of main road = $-\dfrac{3}{4}$. The product of the slope of the main road and the slope of either of the new roads is –1.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• calculate slopes or distances in question 1 • find one completion point for the parallelogram in question 2, but with little or no justification • solve question 3, if given assistance in setting up the axes in the graph • draw the perpendicular roads and calculate the slopes for question 4 • determine the *x*- and *y*-intercepts to the nearest whole number in question 4, part a.	• calculate slopes and distances in question 1, and link these slopes and distances to the properties of a right-angled triangle • find one or more completion points for the parallelogram in question 2, supporting the answer(s) either with an accurate drawing or by linking line slopes and/or lengths to the properties of the parallelogram • solve question 3 • explain why the roads are parallel in question 4 • determine the *x*- and *y*-intercepts in question 4, part a, using interpolation.

STANDARDS IN LINEAR FUNCTIONS

GENERAL OUTCOMES

- Examine the nature of relations with an emphasis on functions.
- Represent data, using linear function models.
- Apply line-fitting and correlation techniques to analyze experimental results.

SPECIFIC OUTCOMES

5.1 Rewrite linear expressions in terms of the dependent (responding) variable. [C, R, V]

5.2 Determine the following characteristics of the graph of a linear function, given its equation in any of the forms $y = mx + b$, $y - y_1 = m(x - x_1)$, $Ax + By + C = 0$, $Ax + By = C$:
- intercepts
- slope
- domain
- range.

[PS, V]

5.3 Determine the equation of a line, given information that uniquely determines the line. [PS, V]

5.4 Use variation and arithmetic sequences as applications of linear functions (use of algebraic and technological means is appropriate). [CN, PS, V]

5.5 Determine the equation of a line of best fit, using:
- estimate of slope and one point
- median–median method
- least squares method with technology.

[CN, PS, T, V]

5.6 Use best-fit linear equations and their associated graphs to make predictions and solve problems. [C, CN, PS, T, V]

5.7 Explain the significance of the parameters a and b in the best-fit equation $y = ax + b$. [C, CN, R, V]

5.8 Use technological devices to determine the correlation coefficient r. [T]

5.9 Interpret the correlation coefficient r and its limitations for varying problem situations, using relevant scatterplots. [C, R, V]

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

5.1 Rewrite linear expressions in terms of the dependent (responding) variable. [C, R, V]

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and	[T] Technology
Mental Mathematics	[V] Visualization

Notes:

- Alternative names for the independent variable include *manipulated* and *input*.
- Alternative names for the dependent variable include *responding* and *output*.
- Students should be encouraged to rewrite equations into the $y = mx + b$ form rather than the other $y =$ forms.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• distinguish responding and manipulated variables in a given context • isolate the responding variable in linear expressions where the coefficients are integers.	• isolate the responding variable in linear expressions where the coefficients are rational numbers.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

5.1   Rewrite linear expressions in terms of the dependent (responding) variable. [C, R, V]

[✓]   Conceptual
[   ]   Procedural
[✓]   Problem-solving

Question:

1.   Isolate the dependent variable y in the expression $5x - y = 20$.

Solution:

1. $5x - y = 20$
$-y = 20 - 5x$
$y = 5x - 20$

Question:

2.   Isolate the dependent variable y in the expression $2x + 3y = 24$.

Solution:

2. $2x + 3y = 24$
$3y = -2x + 24$
$y = -\frac{2}{3}x + 8$

Question:

3.   A family has a $3000 budget for a trip. They estimate costs of $300 per day for each day of the trip and 20¢ for every kilometre of the trip. They want to know how far they can travel for different times spent on the trip.
    a.   What are the manipulated and responding variables in this context?
    b.   Write an equation for the cost of the trip in the form $ad + bt = c$, where d is the number of kilometres and t the number of days.
    c.   Rewrite the equation to isolate the responding variable and use this equation to calculate the distance travelled on an 8-day trip.

(continued)

[✓]		Conceptual
[ ]		Procedural
[✓]		Problem-solving

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

5.1 Rewrite linear expressions in terms of the dependent (responding) variable. [C, R, V]

(continued)

Solution:

3. a. Manipulated: time spent on trip
   Responding: distance travelled on trip.

   b. $ad + bt = c$
      $0.20d + 300t = 3000$

   c. Isolate d in $0.20d + 300t = 3000$
      $0.20d = -300t + 3000$
      $$d = \frac{-300t}{0.20} + \frac{3000}{0.20}$$
      $d = -1500t + 15000$
      If $t = 8$, then $d = -1500(8) + 15000$
      $\qquad\qquad\qquad = -12000 + 15000$
      $\qquad\qquad\qquad = 3000$
      An 8-day trip can be 3000 km long.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• solve questions 1 and 2 • identify the responding variable in question 3, part a • write a relevant equation in question 3, part b.	• solve question 3, part c.

<table>
<tr><td colspan="2">

General and Specific Outcomes

</td><td colspan="2">

[C]   Communication   [PS] Problem Solving
[CN] Connections   [R]   Reasoning
[E]   Estimation and   [T]   Technology
      Mental Mathematics [V]   Visualization

</td></tr>
</table>

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

5.2   Determine the following characteristics of the graph of a linear function, given its equation in any of the forms $y = mx + b$, $y - y_1 = m(x - x_1)$, $Ax + By + C = 0$, $Ax + By = C$:
- intercepts
- slope
- domain
- range.
[PS, V]

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 4.6.
- It is appropriate to emphasize the mathematical process of technology in this specific outcome.
- Students must be able to rewrite equations into the $y = mx + b$ form and to use graphing tools in their analysis.
- Students may express domain and range in words rather than in formal mathematical notation.
- The concepts of domain and range can be related to everyday situations that have restrictions or boundaries; e.g., negative time is not meaningful, and there are limits on how high a person can throw a ball.
- It is appropriate to introduce both algebraic and technological means for solving problems.
- Emphasize the meaning of slope and intercepts in problem solving.
- Alternative names for the independent variable include *manipulated* and *input*.
- Alternative names for the dependent variable include *responding* and *output*.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- graph an equation, but may make errors in choosing the dependent and independent variables - identify the slope and y-intercept, given the equation in slope–intercept form - determine the domain and range - manipulate a given equation into slope–intercept form.	- graph an equation, correctly identifying dependent and independent variables - explain the meaning of the slope and y-intercept in a given situation - explain restrictions on domain and range.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

5.2 Determine the following characteristics of the graph of a linear function, given its equation in any of the forms $y = mx + b$, $y - y_1 = m(x - x_1)$, $Ax + By + C = 0$, $Ax + By = C$:
- intercepts
- slope
- domain
- range.
[PS, V]

[✓] Conceptual
[ ] Procedural
[✓] Problem-solving

Question:

1. A basketball team is hosting a banquet. The caterer charges $10 per person plus a set-up fee of $200. The equation $C = 10n + 200$ represents the cost of the banquet.
   a. Sketch the graph, with the cost as a function of the number of people.
   b. Determine the cost for 30 people attending.
   c. If the total cost is to be no more than $700, how many people can attend?
   d. Determine the slope of the graph, and explain the significance of the slope in this situation.
   e. State the y-intercept, and explain the significance of the y-intercept in this situation.
   f. State the domain and range of the function, and explain why there may be restrictions on both.

Solution:

1. a.   **Cost of Basketball Team's Banquet**
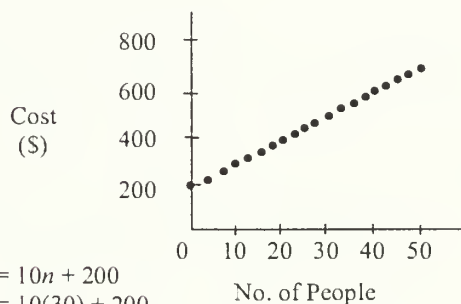
b. $C = 10n + 200$
$C = 10(30) + 200$
$C = 300 + 200$
$C = 500$
The cost would be $500.
Alternatively, use table or calculate value features on a graphing calculator to determine C and n.

c. $700 = 10n + 200$
$500 = 10n$
$n = 50$
The maximum number of people is 50.

(continued)

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

5.2 Determine the following characteristics of the graph of a linear function, given its equation in any of the forms $y = mx + b$, $y - y_1 = m(x - x_1)$, $Ax + By + C = 0$, $Ax + By = C$:
- intercepts
- slope
- domain
- range.
[PS, V]

Sample Task

[✓] Conceptual
[   ] Procedural
[✓] Problem-solving

(continued)
Solution:

1. d. slope = 10 or \$10/person
      slope is cost/person
   e. y-intercept = 200
      y-intercept is the set-up fee for the banquet
   f. Domain : 0, 1, 2, ... to capacity of hall
      Range : 200, 210, 220, ... until hall is full
      There are domain restrictions, because you can never have a negative or fractional number of people, or more people than the banquet hall can hold. There are range restrictions, because the cost goes up in steps of \$10 from \$200 until the hall is full.

Descriptions of Student Performance (Related to Sample Task)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- graph the function - interpolate and extrapolate from the graph - determine the slope and the y-intercept - determine the domain and range as n = whole numbers and C is greater than \$200.	- explain the significance of the slope and y-intercept - restrict the domain to the capacity of the hall, and determine the range as 200, 210, 220, ... - explain the restrictions on the domain and range.

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

5.3 Determine the equation of a line, given information that uniquely determines the line. [PS, V]

[C]	Communication
[CN]	Connections
[E]	Estimation and Mental Mathematics
[PS]	Problem Solving
[R]	Reasoning
[T]	Technology
[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 3.5.
- Students are not expected to solve linear systems in two variables; problems must be solvable without using systems.
- Linear systems are covered in Applied Mathematics 20.
- It is critical for students to be able to manipulate standard equations into the y = form.
- It is appropriate to introduce linear regression, using a graphing calculator to determine the equation of a line.
- Students should sketch the line from the graphing calculator window as part of the solution.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- identify the slope and y-intercept from the equation $y = mx + b$ - write an equation in the form $y = mx + b$, given the slope and y-intercept - differentiate between the slope–intercept form and other forms of linear equations - manipulate a linear equation into any $y =$ form, that can be graphed - recognize the equations of horizontal and vertical lines - determine the equation of a line, in slope–intercept form, given a point and the slope - determine the equation of a line, given a graph of the line with a labelled point on the line - plot points, determine slope, and determine an equation in the form $y = mx + b$.	- determine the equation of a given line on grid paper without labelled points - manipulate a linear equation into the slope–intercept form, which clearly identifies the slope and y-intercept - solve real-world problems related to linear equations.

[   ]   Conceptual
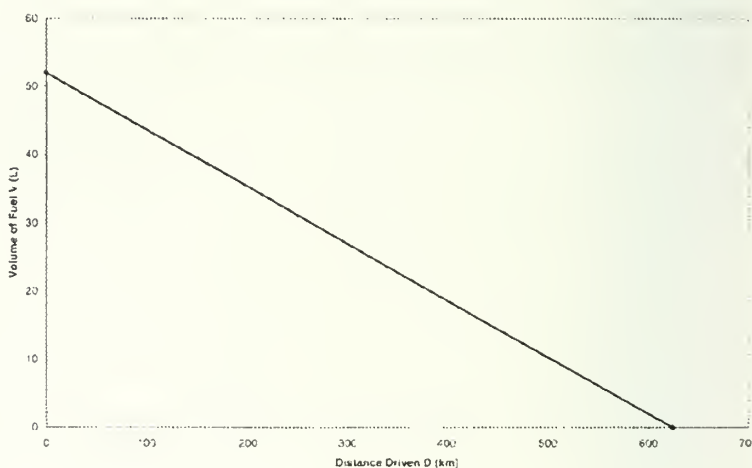[✓]   Procedural
[   ]   Problem-solving

Question:

1. A car with a full gas tank of 52 L is driven until the tank is empty. If the car travelled 624 km, what is the equation that describes the number of litres used per kilometre?

Solution:

1.

Fuel Remaining as a Function of Distance

$$m = \frac{52 - 0}{0 - 624}$$

$$m = \frac{52}{-624}$$

$$m = -\frac{1}{12}$$

$$V = -\frac{1}{12}D + 52$$

where V is volume in L
and D is distance in km.

Question:

2. A local fast food outlet has 150 L of ketchup on Sunday. They typically use 5.0 L of ketchup each day. In order to monitor their supply of ketchup, determine an equation to calculate how much ketchup is left after t days.

Solution:

2. Slope = volume used per day = –5 L/day
    Starting value = 150 L
    $\therefore V(t) = 150 - 5t$

(continued)

General and Specific Outcomes	[ ] Conceptual
	[✓] Procedural
	[✓] Problem-solving

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

5.3   Determine the equation of a line, given information that uniquely determines the line. [PS, V]

(continued)
Question:

3.   Two odometer readings are taken when testing a new car. After 2 hours, 185 km have been covered and after 5 hours, 425 km have been covered. What is the equation that describes the number of kilometres covered as a function of time? Express the equation in appropriate functional notation. What do the values of a and b represent?

Solution:

3.   The co-ordinates are (2, 185) and (5, 425)

$$m = \frac{425 - 185}{5 - 2} \qquad\qquad y = mx + b$$

$$m = \frac{240}{3} \qquad\qquad 185 = 80(2) + b$$

$$m = 80 \qquad\qquad 185 = 160 + b$$

$$25 = b$$

$D(t) = 80t + 25$; $D(t)$ is the distance (km) covered after a specific time, t hours.
The 80 represents the average speed, 80 km/h, and the 25 represents the initial distance, 25 km.

Question:

4.   The size of a rainforest decreases at a rate of 120 000 hectares per year due to deforestation. The current size is 20 million hectares. Deforestation has occurred at this rate for 35 years. Write an equation that will determine the size of the rainforest at any given time.

Solution:

4.   Slope = –120 000 ha/a and initial value = 20 million hectares

Rainforest Size as a Function of Time
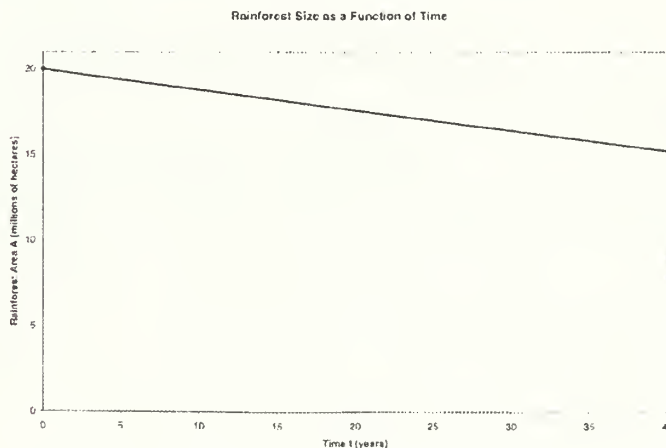
$$A = -12000t + 20\ 000\ 000$$

General and Specific Outcomes

General Outcome

Examine the nature of relations with an emphasis on functions.

Specific Outcome

5.3 Determine the equation of a line, given information that uniquely determines the line. [PS, V]

(continued)
Question:

5.   It costs a candy company $134 to produce 20 gift boxes and $182 to produce 40 gift boxes. Assume the cost per box is the same, regardless of the number produced.
   a.   Write an equation relating costs and the number of gift boxes made.
   b.   How much will the company spend to make 55 gift boxes?
   c.   One day, the company spent $345.20 making gift boxes. How many were produced?
   d.   What do the slope and y-intercept represent?

Solution:

5.   a.   Enter the values (20, 134) and (40, 182) into lists and use the linear regression feature on a graphing calculator to determine the equation of the line: $C = 2.4n + 86$.
   b.   Use the table feature on a graphing calculator to find that it will cost $218 to produce 55 gift boxes.
      OR
      $C = 2.4(55) + 86$
      $C = 132 + 86$
      $C = 218 for 55 boxes.
   c.   Also using the table feature, determine that on this day 108 gift boxes were produced.
      OR
      $$C = 2.4n + 86$$
      $$345.20 = 2.4n + 86$$
      $$345.20 - 86 = 2.4n$$
      $$n = \frac{259.20}{2.4} = 108$$
   d.   The slope represents the cost for one gift box. The y-intercept represents the minimum start-up costs for the process.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• write the equation of the line for question 1 and question 2 • express a straight line in slope–intercept form in question 3 • attempt a solution to question 5, parts a, b and c, but may reverse the dependent and independent variables.	• identify the coordinates of another point on the line in question 4, and determine the equation algebraically or with technology • provide a complete and correct solution to all parts of question 5.

General and Specific Outcomes

General Outcome

Represent data, using linear function models.

Specific Outcome

5.4 Use variation and arithmetic sequences as applications of linear functions (use of algebraic and technological means is appropriate). [CN, PS, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and	[T]	Technology
	Mental Mathematics	[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 4.7.
- Partial variation is included in this specific outcome.
- It is appropriate to introduce both algebraic and technological means for solving problems.
- There is no need to emphasize the specific terminology associated with direct variation.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- determine the equation for a direct variation situation - determine the equation for a partial variation situation - recognize that direct and partial variation situations are represented by linear functions - graph direct and partial variation situations - determine x- and y-intercepts from a graph - determine the numerical value of slope; and interpret slope in obvious contexts, such as speed - make a table of values - interpolate graphs to make estimates.	- explain the significance of the x- and y-intercepts - interpret slope in a wider range of contexts, using slope-appropriate units - write the equation, based on a graph - write the equation, without using a table of values or a graph - extrapolate graphs to make estimates.

General and Specific Outcomes

General Outcome

Represent data, using linear function models.

Specific Outcome

5.4 Use variation and arithmetic sequences as applications of linear functions (use of algebraic and technological means is appropriate). [CN, PS, V]

[✓] Conceptual
[   ] Procedural
[✓] Problem-solving

Question:

1. At a certain time of the day, the length of the shadow of a tree varies directly as the height of the tree. A 3.0 m tree casts a shadow of 1.2 m.
   a. Determine the equation of the variation, and graph the function.
   b. Determine the length of a shadow of a 5.0 m tree.
   c. Determine the height of a tree that produces a shadow that is 6.8 m long.
   d. Find the *x*- and *y*-intercepts, and explain the significance of your findings.

Solution:

1. a. $L = kh$
      When $h = 3$, $L = 1.2$, so $k = 0.4$ and the equation is $L = 0.4h$.

Length of Shadow versus Height of Tree
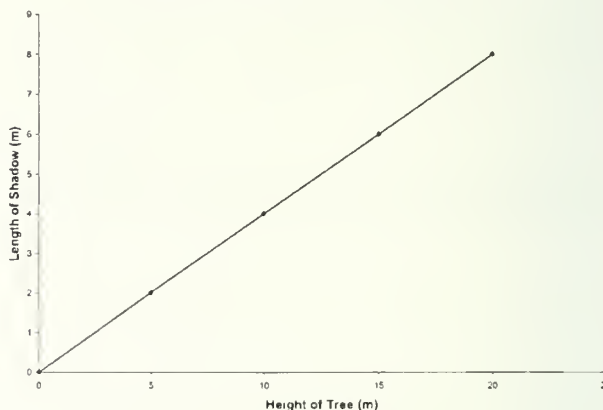
   b. $L = (0.4)(5)$
      $L = 2$
      The length of the shadow is 2 m.
   c. $6.8 = 0.4h$
      $h = 17$
      The height of the tree is 17 m.
   d. *x*-intercept = (0, 0)
      y-intercept = (0, 0)
      The *x*- and *y*-intercepts are the same point, which is the origin. This means that, when the tree has no height, no shadow will be produced.

(continued)

General and Specific Outcomes

General Outcome

Represent data, using linear function models.

Specific Outcome

5.4 Use variation and arithmetic sequences as applications of linear functions (use of algebraic and technological means is appropriate). [CN, PS, V]

(continued)
Question:

2. A vehicle's fuel tank has a maximum capacity of 90 L. Gasoline consumption is rated at 15 L/100 km for the vehicle.
   a. Make a table of values for the distance driven and the amount of fuel left in the tank.
   b. Write the equation, using the amount remaining in the tank as a function of the distance driven.
   c. Graph the function.
   d. Use the graph to find the distance travelled if 50 L of fuel remain in the tank; verify this distance, using your equation.
   e. What distance has been covered when the fuel tank is empty, and how does this relate to the graph?
   f. Explain the significance of the y-intercept.
   g. Explain the significance of the slope.

Solution:

2. a.

Distance (km)	0	100	200	300	400	500	600
Gasoline (L)	90	75	60	45	30	15	0

   b. Using linear regression; $g = -0.15d + 90$.
   Algebraic solution:
   $g = md + b$
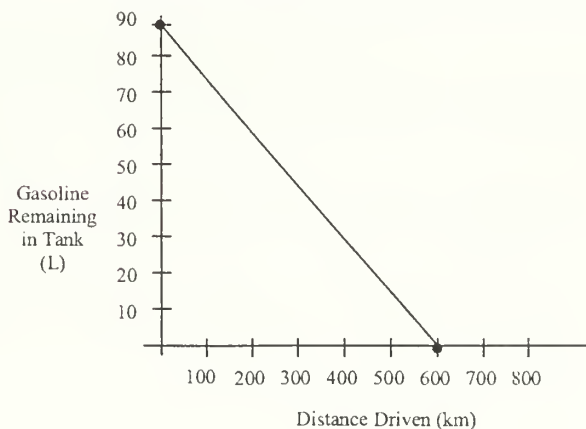 $$m = \frac{75 - 90}{100 - 0}$$
   $$= \frac{-15}{100}$$
   $$= -0.15$$
   $b = 90$ (from table)
   $\therefore g = -0.15d + 90$

   c.

Gasoline Remaining in Tank versus Distance Driven

Distance Driven (km)

(continued)

General and Specific Outcomes	Sample Tasks

General and Specific Outcomes

General Outcome

Represent data, using linear function models.

Specific Outcome

5.4 Use variation and arithmetic sequences as applications of linear functions (use of algebraic and technological means is appropriate). [CN, PS, V]

Sample Tasks

[✓] Conceptual
[   ] Procedural
[✓] Problem-solving

(continued)
Solution:

2. d. Distance = 260 km (from graph)

$$50 = 0.15d$$
$$-40 = 0.15d$$
$$-4\,000 = -15d$$
$$d = 267 \text{ km (algebraically)}$$

  e. 600 km—this is represented by the x-intercept of the graph.
  f. The y-intercept represents the maximum capacity of the tank.
  g. The slope represents the gasoline consumption in L/km.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- find the equation in question 1, part a - draw the graph and find the values in question 1, parts a through c - find the intercepts in question 1, part d - make a table of values and draw the graph in question 2 - read the graph in question 2 to find the values for parts d and e.	- explain the significance of the intercepts in question 1, part d - find the equation in question 2, part b - use the equation to verify the distances in question 2, parts d and e - explain the significance of the slope and both intercepts in question 2.

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.5 Determine the equation of a line of best fit, using:
- estimate of slope and one point
- median–median method
- least squares method with technology.

[CN, PS, T, V]

Notes:

- The principles of the median–median method have to be demonstrated. However, students are expected to determine median–median equations, using technology.
- There is no requirement for students to know the principles of the least squares method.
- Utility 14 in Addison-Wesley *Applied Mathematics 10* provides instructions on how to determine the equation of a line of best fit, using the median–median method.
- Tutorial 6.8 in Addison-Wesley *Applied Mathematics 10* provides instructions on how to determine the equation of a line of best fit, using the least squares method.
- The median–median method should be used when the data set contains an extraneous point.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- generate a scatterplot, using technology and paper and pencil, when given a set of data or when own data is collected - use technology to draw a line of best fit, using the median–median method and the least squares method - determine the equation of the line of best fit, using the slope–intercept method, the median–median method with technology, and the least squares method with technology	- evaluate the validity of the line of best fit with respect to the real-life data - compare the lines of best fit produced by various methods.

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.5 Determine the equation of a line of best fit, using:
- estimate of slope and one point
- median–median method
- least squares method with technology.
[CN, PS, T, V]

Question:

1. The table below gives the populations and areas of some small cities in Alberta in 1997.

City	Area (km^2)	Population
Airdrie	22	15 946
Camrose	27	13 728
Drumheller	25	6 587
Fort Saskatchewan	46	12 408
Grande Prairie	42	31 140
Leduc	26	14 305
Lethbridge	122	66 035
Lloydminster	41	19 324
Medicine Hat	120	45 892
Red Deer	60	60 075
Spruce Grove	27	14 271
Wetaskiwin	17	10 959

a. Graph this data as a scatterplot. Place area on the *x*-axis and population on the *y*-axis.
b. Draw a best-fit line. Use the slope and one point to find the equation of this best-fit line.
c. Use technology to find the equation of the line of best fit, using the median–median method.
d. Use technology to find the equation of the line of best fit, using the least squares method.
e. Compare the two equations.

(continued)

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.5 Determine the equation of a line of best fit, using:
  • estimate of slope and one point
  • median–median method
  • least squares method with technology.
  [CN, PS, T, V]

Sample Task

[✓] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)
Solution:

1. a.

Populations of Cities as a Function of Area
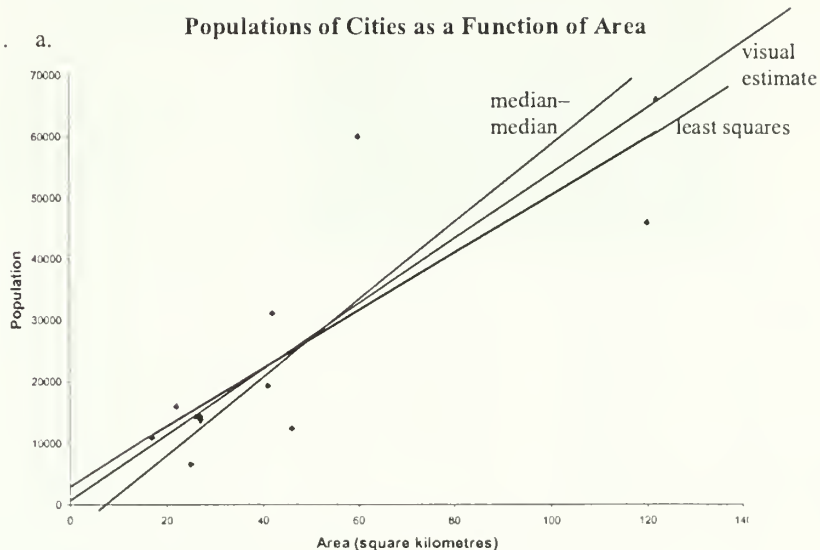
b. slope $= \dfrac{67000 \text{ persons}}{120 \text{ km}^2}$

$= 558 \dfrac{\text{persons}}{\text{km}^2}$

c. Median–Median
$y = 606.8x - 2362.8$ $p = 606.8a - 2362.8$
$p = 607a - 2360$

d. Least Squares
$y = 470.2x + 3360.2$ $p = 470.2a + 3360.2$
$p = 470\,a + 3360$

e. The median–median equation has a higher slope and a lower vertical intercept than the least squares equation.

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• provide solutions to parts a through d.	• compare the two equations in solving part e.

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.6 Use best-fit linear equations and their associated graphs to make predictions and solve problems. [C, CN, PS, T, V]

[C]   Communication   [PS] Problem Solving
[CN] Connections   [R] Reasoning
[E]   Estimation and   [T] Technology
        Mental Mathematics   [V] Visualization

Notes:

- Students must have practice in making interpretations of the line of best fit when solving problems.
- Interpolations are easier to use than extrapolations.
- Interpolations are less risky than extrapolations.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- make inferences from the data and equation of the line - interpolate and extrapolate points from the line of best fit when the view screen does not require any manipulation.	- extrapolate points from the line of best fit when the domain and range must be manipulated on the view screen - evaluate the validity of the line of best fit with respect to the real-life data - compare the lines of best fit produced by various methods.

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.6 Use best-fit linear equations and their associated graphs to make predictions and solve problems. [C, CN, PS, T, V]

Sample Task

[✓] Conceptual
[✓] Procedural
[✓] Problem-solving

Question:

1. The table below gives the populations and areas of some small cities in Alberta in 1997.

City	Area (A) (km^2)	Population (N)
Airdrie	22	15 946
Camrose	27	13 728
Drumheller	25	6 587
Fort Saskatchewan	46	12 408
Grande Prairie	42	31 140
Leduc	26	14 305
Lethbridge	122	66 035
Lloydminster	41	19 324
Medicine Hat	120	45 892
Red Deer	60	60 075
Spruce Grove	27	14 271
Wetaskiwin	17	10 959

The median–median best-fit line is $N = 607A - 2360$ and the least-squares best-fit line is $N = 470A + 3360$.

a. Compare the two equations.
b. If the population of Fort McMurray is 33 078, predict the area using the equations provided.
c. The area of Calgary is 721 km^2. Predict the population using the equations provided.
d. Edmonton has an area of 700 km^2 and a population of 616 306. Does Edmonton fit either model? Explain.

(continued)

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.6   Use best-fit linear equations and their associated graphs to make predictions and solve problems. [C, CN, PS, T, V]

(continued)
Solution:

1.   a.   The median–median equation has a higher slope and a lower vertical intercept than the least-squares equation.

      b. $N = 470.2A + 3360.2$
            Fort McMurray's population, if the least-squares equation is used, can be represented as:
            $470.2\ A + 3360.2$; therefore,
            $33\ 078 = 470.2\ A + 3360.2$
            $A = 63.2$
            Or use the table feature of a calculator.
            Area is predicted as 63.2 km^2.
            Using the median–median equation
            $N = 607A - 2360$,
            so $33078 = 607A - 2360$
                or $A = \frac{33078 + 2360}{607}$
                    $= 58.4$ km^2

      c.   Calgary's population is:
            $470.2(721) + 3360.2 = 342\ 374.4$
            The population is predicted at 343 000, using the least-squares equation.
            Using the median–median equation
            $N = 607(721) - 2360$
              $= 435\ 300$

      d.   Edmonton (700, 616 306)
            Median–Median   Least-Squares
            $N = 606.8\ A - 2362.8$ $N = 470.2\ A + 3360.2$
            $N = 606.8(700) - 2362.8$ $N = 470.2(700) + 3360.2$
            $N = 422\ 400$ $N = 332\ 500$

            The median–median model is closer, but neither predicts an acceptable value for large cities. Population is much denser in the large cities.

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• provide solutions to parts b and c • provide numerical estimate in part d.	• compare the two equations in part a • explain why the model fails for Calgary and Edmonton.

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.7   Explain the significance of the parameters a and b in the best-fit equation $y = ax + b$. [C, CN, R, V]

Notes:

- It is appropriate to expect students to make interpretations of the line of best fit when solving problems.
- It is usual that the slope a has a natural interpretation.
- In many cases the intercept b does not have a real-life interpretation.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• make inferences from the data and equation of the line • relate the slope and y-intercept to the real-life context of the investigation	• evaluate the validity of the line of best fit with respect to the real-life data • compare the lines of best fit produced by various methods.

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.7 Explain the significance of the parameters a and b in the best-fit equation $y = ax + b$. [C, CN, R, V]

Sample Task

Question:

1. The table below gives the populations and areas of some small cities in Alberta in 1997.

City	Area (A) (km^2)	Population (N)
Airdrie	22	15 946
Camrose	27	13 728
Drumheller	25	6 587
Fort Saskatchewan	46	12 408
Grande Prairie	42	31 140
Leduc	26	14 305
Lethbridge	122	66 035
Lloydminster	41	19 324
Medicine Hat	120	45 892
Red Deer	60	60 075
Spruce Grove	27	14 271
Wetaskiwin	17	10 959

The median–median best-fit line is $N = 607A - 2360$ and the least-squares best-fit line is $N = 470A + 3360$.

a. What does the slope represent?
b. Which city has a population that does not fit either model? Justify your choice.
c. What might the value of a represent in the least-squares model?
d. What might the horizontal intercept in the median–median model represent?

Solution:

1. a. The slope represents the population per square kilometre or the population density.

    b. Red Deer has a population density of 1001 persons/km^2. Most other cities have densities nearer to 600 persons/km^2.

    c. This value of 3360 is the population at zero area. It might represent the minimum population for city status.

    d. The horizontal intercept of $\frac{2363}{607}$, or 3.89 km^2, may represent the area of roads and open spaces.

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• provide a solution to part a • identify Red Deer as having a higher population density in part b • find a numerical value in part c.	• provide an explanation in part c • provide both a numerical value and an explanation in part d.

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

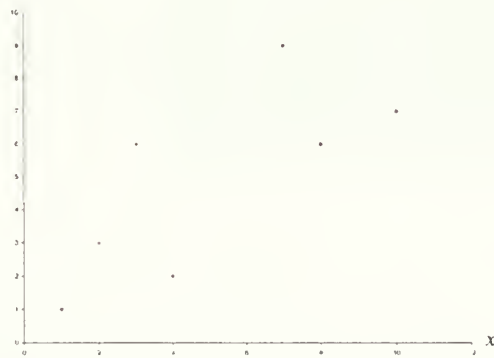5.8 Use technological devices to determine the correlation coefficient *r*. [T]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and	[T]	Technology
	Mental Mathematics	[V]	Visualization

Notes:

- There should be no use of the formula for determining the correlation coefficient, whether calculated manually or by spreadsheet. The value of *r* is found as part of the diagnostics when computing the least squares regression line on a graphing calculator.
- Tutorial 6.9 in Addison-Wesley *Applied Mathematics 10* provides direction for determining correlation coefficients using the TI–83 calculator.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• calculate *r*, using technology • make one generalization regarding the value of *r*.	• make many generalizations regarding the value of *r* • explain the significance of the sign of *r*.

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.8   Use technological devices to determine the correlation coefficient *r*. [T]

[✓]   Conceptual
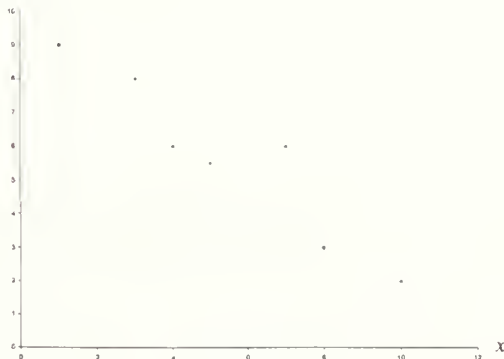[✓]   Procedural
[   ]   Problem-solving

Question:

1.   For parts a through d, do the following:

- Enter the set of data in your calculator.
- Draw the scatterplot for the data set.
- Find the correlation coefficient *r* for the set.

a.

x	y
1	1
2	3
3	6
4	2
7	9
8	6
10	7

b.

x	y
1	2
1.5	3
2	4
2.5	5
4	8
4.5	9
5	10

c.

x	y
1	9
3	8
4	6
5	5.5
7	6
8	3
10	2

d.

x	y
1	5
1	18
3	2
4	6
7	5
8	12
10	1

e.   Compare the appearances of each graph to the value of *r* for the data set. What is the connection between how close a set of points is to being linear and its correlation coefficient?

f.   Which graphs appear to have positive slopes and which appear to have negative slopes? How do these relate to the sign of *r*?

g.   Between what two numbers can the correlation coefficient lie?

(continued)

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.8 Use technological devices to determine the correlation coefficient *r*. [T]

Sample Task

[✓] Conceptual
[✓] Procedural
[   ] Problem-solving

(continued)
Solution:

1. a. *r* = 0.748

b. *r* = 1.00
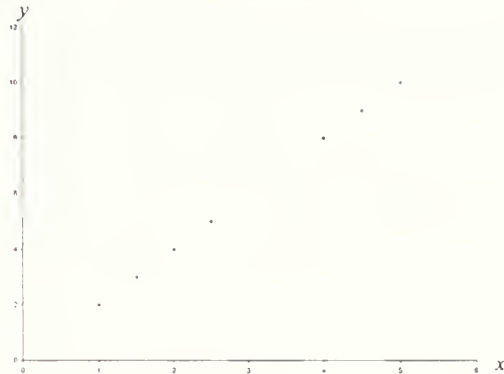
c. *r* = –0.946

(continued)

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.8   Use technological devices to determine the correlation coefficient r. [T]

(continued)
Solution:

1.   d. $r = -0.346$
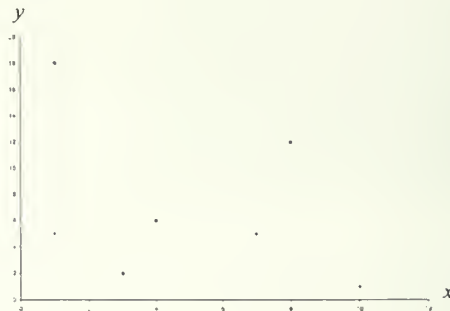
e.   The closer to linear, the nearer r is to $+1$ or -1.
f.   Graphs a and b appear to have positive slopes. Graph c has a negative slope, and graph d could have either. The sign of r is the same as the sign of the slope.
g.   The correlation coefficient can lie between -1 and $+1$.

Descriptions of Student Performance (Related to Sample Task)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• draw scatterplots and find the correlation coefficients for parts a through d • provide partial answers to parts e and f • provide a correct answer to part g.	• provide complete answers to parts e and f.

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.9 Interpret the correlation coefficient r and its limitations for varying problem situations, using relevant scatterplots. [C, R, V]

[C] Communication   [PS] Problem Solving
[CN] Connections   [R] Reasoning
[E] Estimation and   [T] Technology
    Mental Mathematics   [V] Visualization

Notes:

- There should be no use of the formula for determining the correlation coefficient, whether calculated manually or by spreadsheet. The value of r is found as part of the diagnostics when computing the least squares regression line on a graphing calculator.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• sketch scatterplots for different values of r • make inferences about the correlation coefficient r with respect to a scatterplot, or vice versa.	• suggest real-life situations where negative r values occur • make insightful inferences associated with different values of r.

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.9   Interpret the correlation coefficient *r* and its limitations for varying problem situations, using relevant scatterplots. [C, R, V]
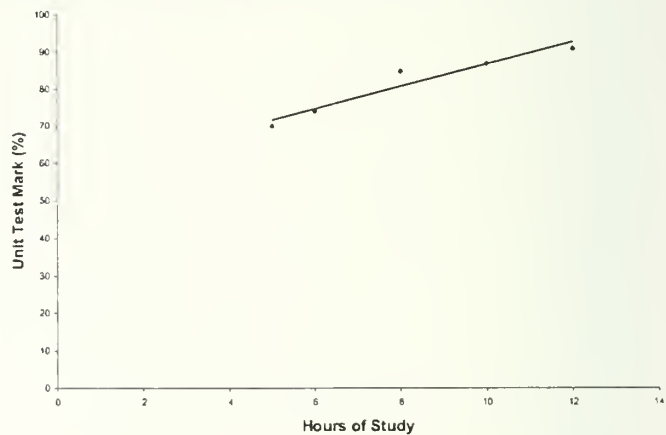
Question:

1.   Jan drew a scatterplot of her marks (%) on mathematics unit tests and the hours spent studying for them. She found the correlation coefficient *r* to be 0.95. She also drew a scatterplot of her friends' final mathematics marks (%) and their physical education final marks (%). She found the correlation coefficient *r* to be 0.17.

Sketch the possible scatterplots describing these situations, with lines of best fit, and explain how the *r* value is related to each scatterplot.
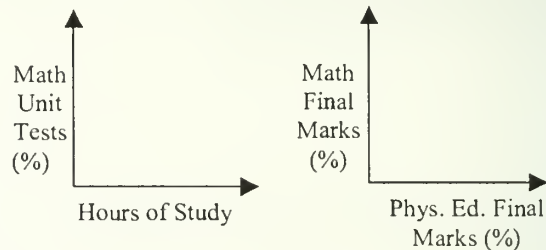
Solution:

1.   **Math Unit Test Marks as a Function of Hours of Study**

(continued)

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.9 Interpret the correlation coefficient *r* and its limitations for varying problem situations, using relevant scatterplots. [C, R, V]

(continued)
Solution:

1. **Math Final Marks as a Function of Phys. Ed. Final Marks**
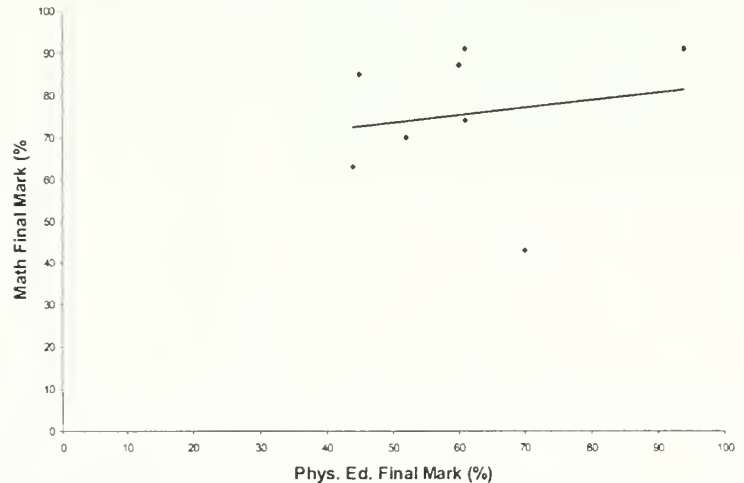
- The positive *r* value indicates that the slope of each line is positive.
- For *r* = 0.95, most of the points are on the line. Very few points are off the line, but even these are close to the line. The *r* value and the graph imply a strong correlation.
- For *r* = 0.17, the points are not close to the line. The *r* value and the graph imply almost no correlation.

Question:

2. a. Sketch two scatterplots with lines of best fit, where the correlation coefficients are 0.75 and –0.75.
   b. What are the similarities and differences between these scatterplots?
   c. Suggest a real-life situation where *r* = –0.75.

(continued)

General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.9 Interpret the correlation coefficient r and its limitations for varying problem situations, using relevant scatterplots. [C, R, V]
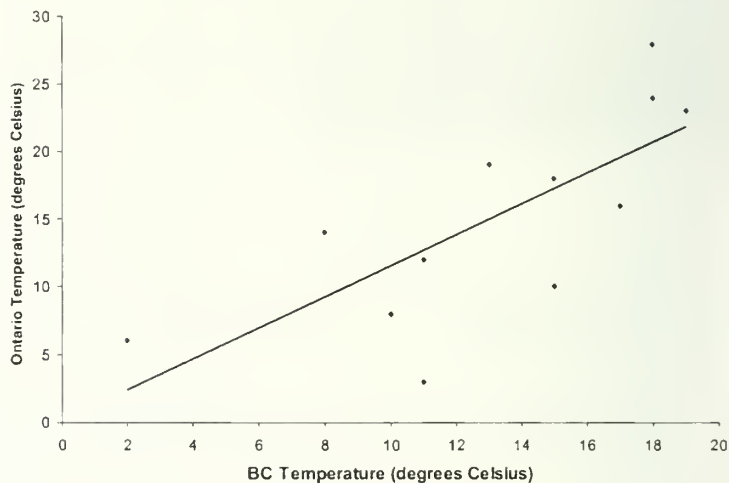
Sample Tasks

[✓] Conceptual
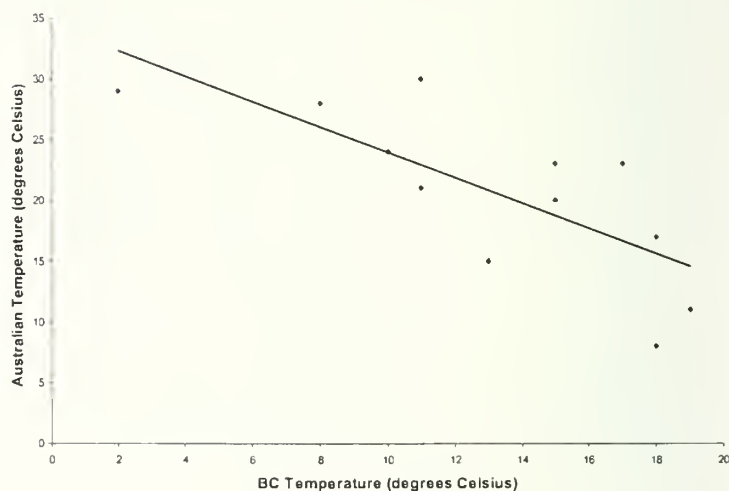[✓] Procedural
[   ] Problem-solving

(continued)
Solution:

2. a.

**Ontario Temperature Related to
BC Temperature on the Same Day
(One Day for each Month)**

**Australian Temperature Related to
BC Temperature on the Same Day
(One Day for each Month)**

(continued)

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General and Specific Outcomes

General Outcome

Apply line-fitting and correlation techniques to analyze experimental results.

Specific Outcome

5.9 Interpret the correlation coefficient r and its limitations for varying problem situations, using relevant scatterplots. [C, R, V]

</td><td>

[✓] Conceptual
[✓] Procedural
[ ] Problem-solving

(continued)
Solution:

2. b. The points for both scatterplots are equally close to the line of best fit.
For $r = 0.75$, the slope of the best fit line is positive; and for $r = -0.75$, the slope of the best fit line is negative.
   c. A possible situation where $r = -0.75$ is the relationship between temperatures for northern and southern hemisphere cities on the same day.

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</table>

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• sketch scatterplots for question 1 and question 2, part a • provide simple answers when comparing r values and scatterplots in question 1 and question 2, part b.	• provide detailed and insightful answers when comparing r values and scatterplots, and provide an example of a real-life situation for question 2, part c.

GENERAL OUTCOME

- Solve problems involving triangles, including those found in 3-D and 2-D applications.

SPECIFIC OUTCOMES

6.1 Solve problems involving two right triangles. [CN, PS, V]

6.2 Extend the concepts of sine and cosine for angles from 0° to 180°. [R, T, V]

6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

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General and Specific Outcomes

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[C]   Communication

[CN] Connections

[E]   Estimation and
       Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T]   Technology

[V] Visualization

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</table>

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.1 Solve problems involving two right triangles. [CN, PS, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and	[T]	Technology
	Mental Mathematics	[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 6.1.
- Students have solved problems involving a single right triangle in Grade 9 Mathematics—Shape and Space (Measurement), specific outcomes 3 and 4.
- An extensive review of the Grade 9 outcomes is required, especially for finding the hypotenuse of a single right triangle.
- Calculators are assumed to be kept running with any multipart calculations. Only final answers are to be rounded. All input data is assumed to be measured. Refer to the Mathematical Conventions Used in the Description of Sample Tasks section of this document.
- Solutions of systems of equations are not part of Applied Mathematics 10.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- solve problems involving two right triangles, where the shared side is a given value - solve two-dimensional problems, if the diagram is given.	- solve problems involving two right triangles, where the shared side is determined from given information and is needed to solve the second triangle - solve complex problems by drawing his/her own diagram.

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.1   Solve problems involving two right triangles. [CN, PS, V]

Question:

1.   A rugby post has three key points: the base *C*, the crossbar *C* and the top *T*. A surveyor at a point *S* measures the distance *SB* as 26.70 m, the distance *SC* as 26.87 m and the distance *ST* as 28.02 m. From this data, calculate the height of *BC* (crossbar height), to the nearest centimetre, and the angles *CSB* and *CST*, to the nearest hundredth of a degree.

Solution:

1.
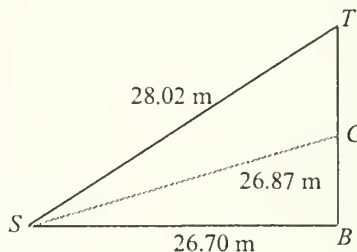
$$BC^2 + 26.70^2 = 26.87^2$$
$$\therefore \ BC = 3.02 \text{ m}$$

$$\cos[\angle TSB] = \frac{26.70}{28.02}$$
$$\angle TSB = 17.6567\ldots°$$
$$\cos[\angle CSB] = \frac{26.70}{26.87}$$
$$= 6.4485\ldots°$$

Angle *CSB* is 6.45°.

So angle *CST* is
        17.6567° − 6.4485°
or 11.21°

The crossbar height is 3.02 m, the angle *CSB* is 6.45°, and the angle *CST* is 11.21°.

(continued)

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.1 Solve problems involving two right triangles. [CN, PS, V]

[   ]   Conceptual
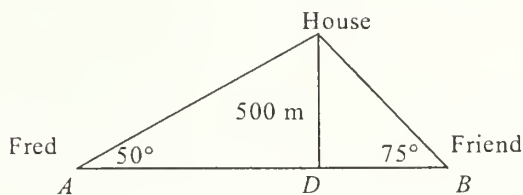[   ]   Procedural
[✓]   Problem-solving

(continued)
Question:

2. Fred and his friend were surveying along a straight road $\overline{AB}$. A house is 500 m from the nearest part of the road. Fred measures the angle to the house to be 50°, while his friend measures the angle to the house to be 75°. How far apart, to the nearest metre, are Fred and his friend?

Solution:
2.

House

500 m

Fred $\overset{\displaystyle 50°}{}$ 75° Friend

A   D   B

$$\tan 50° = \frac{500}{AD} \qquad\qquad AD = 419.55$$

$$\tan 75° = \frac{500}{DB} \qquad\qquad DB = 133.97$$

$$AB = 553.52 \text{ m} = 554 \text{ m}.$$

Fred and his friend are 554 m apart.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• calculate the crossbar height in question 1, if the diagram is given • calculate angle *CSB* in question 1, if the diagram is given • solve question 2, if the diagram is given.	• solve both questions, even if the diagrams are not given.

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.2 Extend the concepts of sine and cosine for angles from 0° to 180°. [R, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and	[T]	Technology
	Mental Mathematics	[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 6.2.
- This specific outcome requires less time compared to specific outcomes 6.1 and 6.3.
- Students should be able to use a calculator for angles up to 180°.
- Students should be able to explain informally why the sine is positive and the cosine negative for angles between 90° and 180°.
- Exact values for sines and cosines are not part of applied mathematics.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• use a calculator to find the values of the sine and cosine of angles from 0° to 180° • use a calculator to find the angle between 0° and 180°, given the value for the cosine of the angle • use a calculator to find the angle between 0° and 90°, given the value for the sine of the angle • draw the graphs of sine and cosine, when calculator windows are given.	• illustrate the use of sine and cosine ratios in a triangle with one angle between 90° and 180° • set the calculator windows for sine and cosine graphs.

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.2　Extend the concepts of sine and cosine for angles from 0° to 180°. [R, T, V]

[✓]　Conceptual
[✓]　Procedural
[　]　Problem-solving

Question:

1.　Use your calculator to determine cos 35°, sin 145° and cos 110°, to four decimal places.

Solution:

1.　0.8191, 0.5736, –0.3420

Question:

2.　Determine angle A for the following.
　　a.　$\cos A = 0.47$
　　b.　$\cos A = -\dfrac{4}{5}$
　　c.　$\sin A = 1$
　　d.　$\sin A = 0.5$

Solution:

2.　a.　61.97°　b. 143.13°　c. 90°　d. 30° or 150°

Question:

3.　For a triangle ABC,
　　$AC^2 = AB^2 + BC^2 - 2(AB) \times (BC) \cos B$
　　For a triangle with $AB = BC = 4$ cm, and angle $B = 120°$, AC is measured as 7.0 cm. What value for cos 120° does this give? How close is this to the calculator value for cos 120°?

Solution:

3.　cos 120° = –0.5313, compared to –0.5000 on the calculator

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• solve question 1 • solve question 2, parts a, b and c • obtain one solution for question 2, part d.	• find the second solution for question 2, part d • solve question 3.

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

Notes:

- This specific outcome is common with Pure Mathematics 10 specific outcome 6.3.
- Neither the sine law nor the cosine law should be derived.
- Calculators are assumed to be kept running with any multipart calculations. Only final answers are to be rounded. All input data is assumed to be measured. Refer to the Mathematical Conventions Used in the Description of Sample Tasks section of this document.
- Angle–side–side cases are optional; these may be covered as part of vectors in Applied Mathematics 30.
- All triangles are assumed to be unique in applied mathematics.
- Students should be provided with the manipulated form of the cosine law: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
- determine any side of a triangle, given the diagram and using either the sine or cosine law provided - determine any acute angle of a triangle, given the diagram and using the sine law provided - determine any angle of a triangle, given the diagram and using the cosine law provided.	- determine any side or angle of a triangle, using the sine or cosine law, whether or not a diagram is given - use sine and cosine laws to solve problems involving more than one triangle.

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

[ ] Conceptual
[✓] Procedural
[✓] Problem-solving
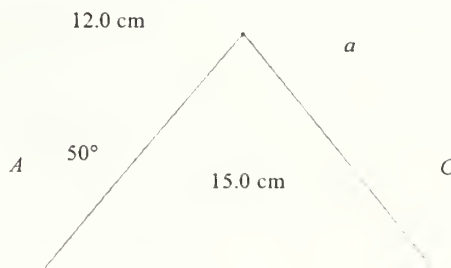
Question:

1. Apply the sine law to solve for side a.
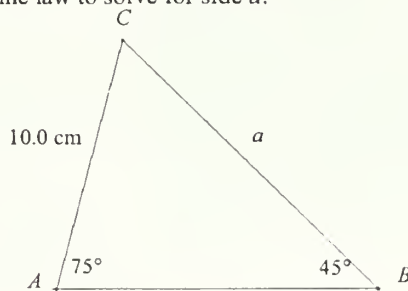
Solution:

1. $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$ or $\dfrac{a}{\sin 75°} = \dfrac{10}{\sin 45°}$; $a = 13.7$ cm

Question:

2. Apply the cosine law to solve for side a to the nearest 0.1 cm.

Solution:

2.
$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$a^2 = (15)^2 + (12)^2 - 2(15)(12) \cos 50°$$
$$a = 11.7 \text{ cm}$$

Question:

3. Solve for angle A to the nearest 0.1°.
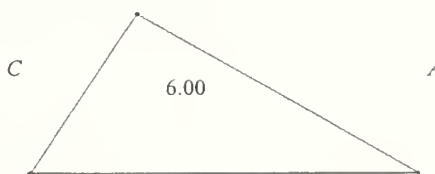
(continued)

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

Sample Tasks

[ ] Conceptual
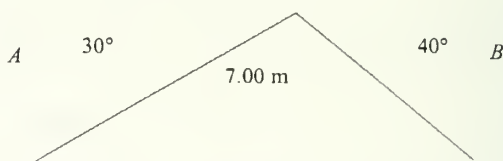[✓] Procedural
[✓] Problem-solving

(continued)
Solution:

3.
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos A = \frac{6^2 + 5^2 - 3^2}{2(6)(5)}$$

$$A = 29.9°$$

Question:

4. James is building a greenhouse. To take advantage of the sunshine, James constructs the roof as illustrated.

Determine the lengths of the roof sections AC and BC, to the nearest 0.1 m, and determine the angle of the roof, C, to the nearest degree.

Solution:

4.
$$C = 180° - (30° + 40°)$$
$$= 110°$$
$$\frac{7.0}{\sin 110°} = \frac{BC}{\sin 30°}$$
$$BC = 3.7 \text{ m}$$
$$\frac{7.0}{\sin 110°} = \frac{AC}{\sin 40°}$$
$$AC = 4.8 \text{ m}$$

(continued)

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(August 2002)

Outcomes with Assessment Standards for Applied Mathematics 10
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General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

(continued)

Question:

5. Three Alberta towns are situated such that they form a triangle. The distance between Medicine Hat and Brooks is 118 km. The distance between Brooks and Taber is 100 km. The distance between Medicine Hat and Taber is 114 km. Find the smallest angle among the highways.

Solution:

5.
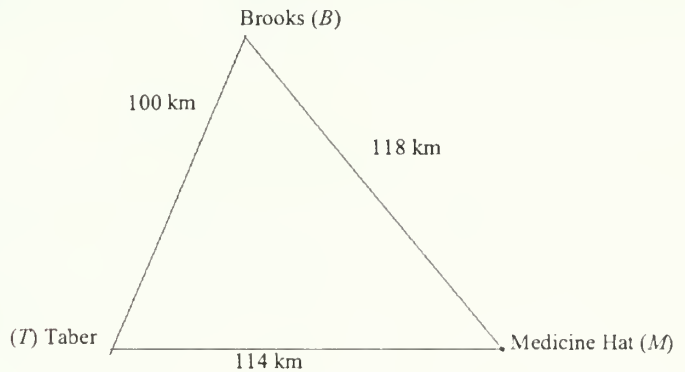
$$\cos B = \frac{118^2 + 100^2 - 114^2}{2(118)(100)}$$

$$B = 62.4°$$

$$\frac{\sin B}{b} = \frac{\sin M}{m}$$

$$\sin M = \frac{m \sin B}{b}$$

$$\sin M = \frac{100 \sin 62.4°}{114}$$

$$M = 51.0°$$

$$T = 180° - (62.4 + 51)$$

$$T = 66.6°$$

The smallest angle is angle M at 51.0°.

(continued)

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

6.3   Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

Sample Tasks

[   ]   Conceptual
[✓]   Procedural
[✓]   Problem-solving

(continued)
Question:

6.   Alice is in a watchtower, and John and Fred are searching for a lost child. The watchtower is 75 m high. Fred radios in that he has found some evidence; he estimates his angle of elevation to the tower to be 12°. Alice radios the information to John. John's estimate of his angle of elevation is 25°. Alice estimates that the angle from Fred to the base of the tower to John is 105°. How far apart, to the nearest metre, are John and Fred?
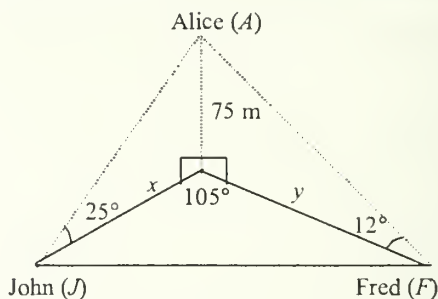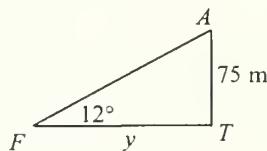

Solution:

6.   Let T represent the bottom of the tower.
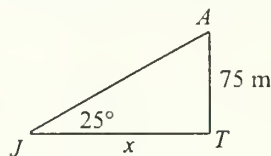
$$\tan 25° = \frac{75}{x}$$
$$x = 160.838019 \text{ m}$$

$$\tan 12° = \frac{75}{y}$$
$$y = 352.8472582 \text{ m}$$

(continued)

General and Specific Outcomes

General Outcome

Solve problems involving triangles, including those found in 3-D and 2-D applications.

Specific Outcome

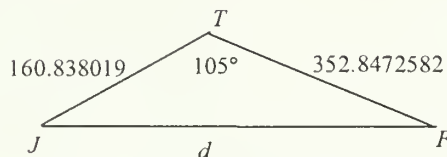6.3 Apply the sine and cosine laws, excluding the ambiguous case, to solve problems. [CN, PS, V]

Sample Tasks

[   ] Conceptual
[✓] Procedural
[✓] Problem-solving

(continued)
Solution:

6.

$$d^2 = x^2 + y^2 - 2(x)(y)\cos 105°$$
$$d = 423.965 \text{ m}$$
$$= 424 \text{ m}$$
where $x = 160.838019$
$y = 352.8472582$

Fred and John are 424 m apart.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the *acceptable standard* can:	A student demonstrating the *standard of excellence* can also:
• solve questions 1, 2 and 3 • solve questions 4 and 5, if the diagrams are supplied.	• solve question 5, translating verbal information into diagram form • solve question 6, if the diagram is supplied.